Math-484 Homework #3

I will finish this homework before 11 am Sep 14 and bring it to class. If I have troubles with my work I may come to the study session on Sep 12, 5-7 pm, 145 Altgeld Hall. If I spot a mathematical mistake I will let the lecturer know as soon as possible.
I will write clearly and neatly as the grader is not an expert in cryptography. I will sign each paper of my work and indicate if I am D14 (4 hours student).

Exercise 1: *(Do I understand the definition of a convex set?)*
Are the follwing sets D in $\mathbb{R}^2$ convex?
(a) $\mathbf{x} = (100, 14) \in \mathbb{R}^2, \mathbf{y} = (15, 24) \in \mathbb{R}^2$.
$D = \{\mathbf{w} \in \mathbb{R}^2 : \mathbf{w} = \lambda\mathbf{x} + (1-\lambda)\mathbf{y},\ \text{where } 0.3 < \lambda \leq 0.7\}$
(b) $D = B((1, 0), 1) \cup (0, 0)$ recall $B(\mathbf{x}, r) = \{\mathbf{w} : ||\mathbf{x} - \mathbf{w}|| < r\}$
(c) $D = B((1, 1), 1) \cup (0, 0)$

Exercise 2: *(Do I understand more than just a* picture of the definition*?)*
Prove Theorem 16 (2.1.4): Let $D \subseteq \mathbb{R}^n$. Then $co(D)$ coincides with the set C of all convex combinations of vectors from D.

Hint:
1) Show that C is a convex set containing D.
2) Show that if B is a convex set containing D then it also contains C.
3) Conclude that $co(D) = C$.

Exercise 3: *(I recall the definition of a convex function.)*
Determine whether the functions are convex, concave, strictly convex or strictly concave on the specified sets:
(a) $f(x) = \ln x$ for $x \in (0, +\infty)$
(b) $f(x) = |x|$ for $x \in \mathbb{R}$
(c) $f(x_1, x_2) = 5x_1^2 + 2x_1x_2 + x_2^2 - x_1 + 2x_2 + 3$ for $(x_1, x_2) \in \mathbb{R}^2$
(d) $f(x_1, x_2) = (x_1 + 2x_2 + 1)^8 - \ln(x_1x_2)^2$ for $\{(x_1, x_2) \in R^2 : x_1 > x_2 > 1\}$
(e) $f(x_1, x_2) = c_1x_2 + c_2/x_1 + c_3x_2 + c_4/x_2$ for $\{(x_1, x_2) \in R^2 : x_1 > 0, x_2 > 0\}$, where c_1, c_2, c_3, and c_4 are positive constants

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Exercise 4: *(I can solve a trickier convex function problem?)*
Let $f(\mathbf{x})$ be defined on set $D = \{\mathbf{x} \in \mathbb{R}^3 : x_1 > 0, x_2 > 0, x_3 > 0\}$ as

$$f(x_1, x_2, x_3) = (x_1)^{r_1} + (x_2)^{r_2} + (x_3)^{r_3}.$$

Show that $f(\mathbf{x})$ is
(a) strictly convex on D if $r_i > 1$ for $i = 1, 2, 3$.
(b) strictly concave on D if $0 < r_i < 1$ for $i = 1, 2, 3$.

Exercise 5: *(More theoretical convex functions. **D14 only**)*
Prove Theorem 17 (2.3.1): Let f be a convex function defined on an open interval $(a, b) \subset \mathbb{R}$. Then f is continuous on (a, b).

Hint:
See page 78, exercise 4. for an outline of the proof.
