

Math-484 Homework #5

I will finish this homework before 11 am Oct 5 and bring it to class. If I have troubles with my work I may come to the study session on Oct 3, 5-7 pm, 145 Altgeld Hall. If I spot a mathematical mistake I will let the lecturer know as soon as possible.

I will write clearly and neatly as the grader is not an expert in cryptography. I will sign each paper of my work and indicate if I am D14 (4 hours student).

Exercise 1: (Do I know how to solve inconsistent system?)

Find the least squares solution of the inconsistent linear system of 6 equations:

$$\begin{array}{rcl} x_1 + x_2 + x_3 = 3 & & x_3 = 1 \\ x_1 + x_3 = 2 & & 2x_1 + 5x_3 = 8 \\ -7x_1 + 8x_2 = 0 & & x_1 + 2x_2 - x_3 = 1 \end{array}$$

Exercise 2: (Can I do understand linear regression?)

Compute the equation of the linear regression line corresponding to the data on the table below:

x	-2	-1	0	1	2	3
y	12	11	8	5	2	-3

Exercise 3: (How much can the solution change?)

Compute and compare solutions of the two systems

$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0.01 \end{pmatrix}.$$

Exercise 4: (How do I compute generalized inverz?)

Compute generalized inverz A^\dagger of

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 3 \end{pmatrix}$$

Exercise 5: (Am I familiar with Gram-Schmidt Process?)

Find orthonormal bases of linear subspaces generated by:

- a) $L_1 = \{(0, 3, 4, 0)^T, (0, 0, 5, 0)^T, (2, 1, 0, 2)^T\}$
b) $L_2 = \{(2, 0, 1, 2)^T, (4, 3, 2, 4)^T, (6, -5, 3, 6)^T, (-4, 2, 4, 2)^T\}$

Exercise 6: (Applications of $(A - G)$). **D14 only**)

Solve the following classical calculus problems by making use of $(A - G)$ inequality.

- a) Find the largest circular cylinder that can be inscribed in a sphere of a given radius.
b) Find the smallest radius r such that a circular cylinder of volume 8 cubic units can be inscribed in the sphere of radius r .
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