Math-484 Homework #9

I will finish this homework before 11 am Nov 16 and bring it to class. If I have troubles with my work I may come to the study session on Nov 14, 5-7 pm, 145 Altgeld Hall. If I spot a mathematical mistake I will let the lecturer know as soon as possible.

Exercise 1: *(Do I really understand lienar programming duality?)*

Prove Farkas Lemma. That is: Let $A \in \mathbb{R}^{m \times n}$ and $\mathbf{b} \in \mathbb{R}^n$. Prove, that the system

$$A^T \mathbf{x} = \mathbf{b}$$

has a solution $\mathbf{x} \geq \mathbf{0}$ if and only if

$$\mathbf{b}^T \mathbf{y} \geq \mathbf{0} \text{ whenever } A\mathbf{y} \geq 0.$$

Hint:

See pages 212 and 213 in the textbook.

Exercise 2: *(Can I use KKT?)*

Apply the Karush-Kuhn-Tucker Theorem to locate all solutions of the collowing convex programs:

$$(P_a) \begin{cases} \text{Minimize } f(x_1, x_2) = e^{-(x_1+x_2)} \\ \text{subject to } e^{x_1} + e^{x_2} \leq 20 \\ x_1 \geq 0 \end{cases} \qquad (P_b) \begin{cases} \text{Minimize } f(x_1, x_2) = x_1^2 + x_2^2 - 4x_1 - 4x_2 \\ \text{subject to } x_1^2 - x_2 \leq 0 \\ x_1 + x_2 \leq 2 \end{cases}$$

Exercise 3: *(Can I solve geometric program?)*

Consider the follwoing geometric program:

$$(GP) \begin{cases} \text{Minimize } f(t_1, t_2) = t_1^{-1} t_2^{-1} \\ \text{subject to } \frac{1}{2}t_1 + \frac{1}{2}t_2 \leq 1 \\ \text{where } t_1 > 0,\ t_2 > 0 \end{cases}$$

a) Convert the resulting program to an eqivalent convex program and solve the resulting program using KKT.
b) Solve the fiven (GP) by using methods of Chapter 5.3.

Exercise 4: *(More geometric programming.)*

Solve the follwoing geometric program:

$$(GP) \begin{cases} \text{Minimize } x^{1/2} + y^{-2}z^{-1} \\ \text{subject to } x^{-1}y^2 + x^{-1}z^2 \leq 1 \\ \text{where } x > 0,\ y > 0,\ z > 0 \end{cases}$$

Exercise 5: *(Can I use the theory a bit?)*

Let $f(x)$ be a differentiable function on $\mathbb{R}$. Suppose x_0 is fixed and there exists a number $\alpha \in \mathbb{R}$ such that

$$f(x) \geq f(x_0) + \alpha(x - x_0)$$

for all $x \in \mathbb{R}$. Show that $\alpha = f'(x_0)$.

Exercise 6: ***(Bonus for D14)***

Let M be a subspace of $\mathbb{R}^n$. From the definitions, it is clear that $M \subseteq (M^\perp)^\perp$. Use the Basic Separation Theorem to show $(M^\perp)^\perp \subseteq M$. Thus giving a proof that $M = (M^\perp)^\perp$.