

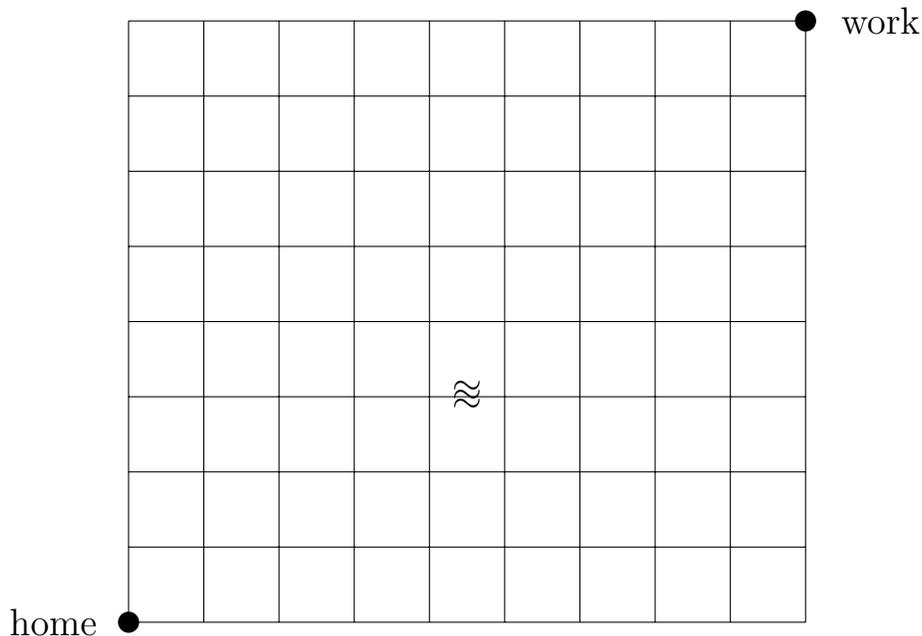
MATH413 HW 3

due **Feb 15** before class

1: (*P. 63, #28*) A secretary works in a building located nine blocks east and eight block north of his home. Every day he walks 17 blocks to work. (See the map that follows.)

(a) How many different routes are possible for him?

(b) How many different routes are possible if the one block in the easterly direction, which begins four block west and three blocks north of his home, is under water (and he can't swim)? (Hint: use subtraction principle)



2: (*P. 83, #5*) Show that if $n+1$ integers are chosen from the set $\{1, 2, \dots, 3n\}$, then there are always two which differ by at most 2.

3: (*P. 83, #8*) Use the pigeonhole principle to prove that the decimal expansion of rational number m/n eventually is repeating. For example,

$$\frac{34,478}{99,900} = 0.345125125125 \dots$$

4: (*P. 83, #10.*) A child watches TV at least one hour each day for seven weeks but, because of parental rules, never more than 11 hours in any one week. Prove that there is some period of consecutive days in which the child watches exactly 20 hours of TV. (It is assumed that the child watches TV for a whole number of hours each day.)

5: (*P. 83, #12.*) Show by example that the conclusion of the Chinese remainder theorem (Application 6) need not hold when m and n are not relatively prime.

6: (*P. 84, #15*) Prove that, for any $n + 1$ integers a_1, a_2, \dots, a_{n+1} , there exist two of the integers a_i and a_j with $i \neq j$ such that $a_i - a_j$ is divisible by n .

7: (*P. 84, #19*) (a) Prove that of any five points chosen within an equilateral triangle of side length 1, there are two whose distance apart is at most $\frac{1}{2}$.

(c) Determine an integer m_n such that if m_n points are chosen within an equilateral triangle of side length 1, there are two whose distance apart is at most $1/n$.