MATH413 HW 8

due **Apr 13 strictly before class**

1: *P.260, #23* Let α be a real number. Let the sequence $h_0, h_1, h_2, \ldots, h_n, \ldots$ be defined by $h_0 = 1$, and $h_n = \alpha(\alpha-1)\cdots(\alpha-n+1), (n \geq 1)$. Determine the exponential generating function for the sequence.

2: Solve (find expression for h_n) the following recurrences:
(a) $h_{-1} = 3, h_0 = 4, h_{n+1} = 4h_n - 3h_{n-1}$
(b) $h_0 = 3, h_1 = 4, h_{n+2} = 4h_n + 2$

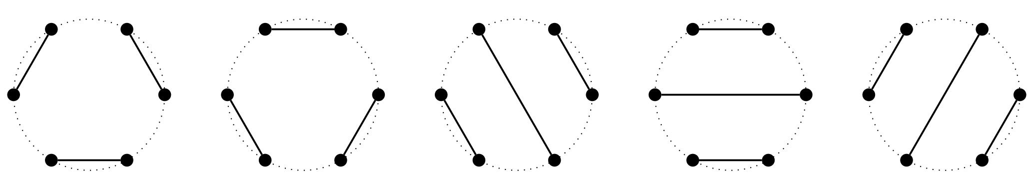
3: *(P.263,#47)* Solve the nonhomogeneous recurrence relation

$$\begin{aligned} h_n &= 4h_{n-1} - 4h_{n-2} + 3n + 1 \\ h_0 &= 1 \\ h_1 &= 2. \end{aligned}$$

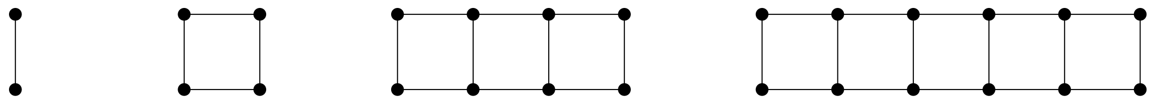
4: Let there be $2n$ points V on a circle in the plane. A *perfect matching* M is a set of segments with endpoints only from V and every point in V is an endpoint of exactly one segment. Note that $|M| = n$ as one segment needs exactly 2 points from V. A matching M in *non-crossing* if the segments are disjoint. Find the number of non-crossing perfect matchings for $2n$ points.

This can be stated in graph theory language as follows. Count the number of perfect matchings of K_{2n} with vertices are vertices of a regular $2n$-gon in the plane such that the edges of the matching do not cross.

Example for $n = 3$ and hence 6 points.



5: Count the number of perfect matchings of points of an earthworm W_{2n} on $2n$ vertices when only segments like in the picture may be used in the matching.



W_2 W_4 W_8 W_{12}

For example W_4 has two perfect matchings (a) and (b):

(a) (b)