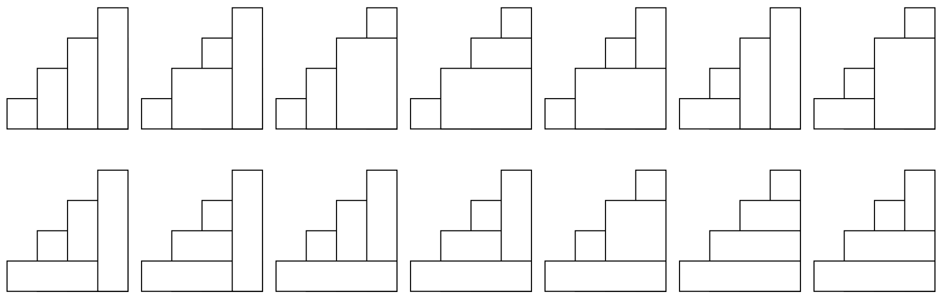
MATH413 HW 9

due **Apr 18 strictly before class**

Solutions without explanation will receive no points.

1: Find the number of possibilities to build stairs of height n using n rectangular bricks. All the possibilities for $n = 4$ are depicted.



2: *P. 315, # 2* Prove that the number of 2-by-n arrays

$$\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \end{bmatrix}$$

that can be made from numbers $1, 2, \ldots, 2n$ such that

$$x_{11} < x_{12} < \cdots < x_{1n}$$

$$x_{21} < x_{22} < \cdots < x_{2n}$$

$$x_{11} < x_{21}, x_{12} < x_{22}, \ldots, x_{1n} < x_{2n},$$

equals the n^{th} Catalan number, C_n.

3: *P. 316, # 12* Prove that the Stirlling numbers of the second $S(n,k)$ kind satisfy the following relations:

(a) $S(n,1) = 1$ for $n \geq 1$

(b) $S(n,2) = 2^{n-1} - 1$ for $n \geq 2$

(c) $S(n,n-1) = \binom{n}{2}$ for $n \geq 1$

(d) $S(n,n-2) = \binom{n}{3} + 3\binom{n}{4}$ for $n \geq 2$

4: Let $[x]_n = x \cdot (x-1) \cdot (x-2) \cdot (x-3) \cdots (x-n+1)$ and $S(n,k)$ be the Stirling number of the second kind. Show that

$$x^n = \sum_{k=1}^{n} S(n,k)[x]_k.$$

5: *P. 317, # 16* Compute the Bell number B_8.

6: *P. 317, #19* Prove that the Stirling numbers of the first kind satisfy the following formulas:
(a) $|s(n,1)| = (n-1)!$ for $n \geq 1$
(b) $|s(n,n-1)| = \binom{n}{2}$ for $n \geq 1$