MATH413 HW 11 - bonus questions

1: Does there exist a BIBD with parameters $b = 10, v = 8, r = 5$, and $k = 4$?

2: Does there exist a BIBD with parameters $b = 20, v = 18, r = 10$, and $k = 9$?

3: Construct three mutually orthogonal latin squares of order four.

4: Construct three mutually orthogonal latin squares of order seven.

5: Fill the missing entries to get a Latin square

	3	1			2
3	1			2	
1		3	2		
		2	3		1
	2			1	3
2			1	3	

6: Construct a completion of the following Latin rectangle 3×6 into a Latin square of order 6.

1	2	3	4	5	6
5	4	2	6	3	1
6	5	4	1	2	3

7: For $m \leq n$ let the *Latin rectangle* be a matrix $m \times n$ where every entry is in $\{1, 2, \ldots, n\}$, every row is a permutation of all n elements and every column contains m distinct elements. Compute the number of Latin squares of size $2 \times n$.

8: (a) Construct projective plane from $\mathbb{Z}_3$.
(b) Construct as many mutually orthogonal latin squares from the resulting projective plane as you can.

9: Is it possible to construct a projective plane from the following two Latin squares? If yes, construct it.

1	2	3
2	3	1
3	1	2

1	2	3
3	1	2
2	3	1

10: Decide for which n there exists a finite projective plane of order n. Consider only $n \in \{2, 3, 4, 5, 6, 7, 8, 9\}$.
(Note: for $n = 10$ it does not exist - was shown by a computer.)

11: Let (X, L) be a projective plane of order n. A set $Y \subseteq Y$ is called *2-blocking* if every line $p \in L$ contains at least two points from Y.
(a) Find the smallest 2-blocking set for the Fano plane (projective plane of order 2).
(b) Show that for every projective plane of order n construct a 2-blocking set of size at most $3n$.
(c) Show that there is no 2-blocking set of size $2n$ for any projective plane of order n.

12: Let (X, L) be a projective plane of order n. Show that it is possible to label points $x_1, x_2, \ldots, x_n$ and lines $p_1, p_2, \ldots, p_n$ such that $x_i \in p_i$ for every $i \in \{1, \ldots, n\}$.

13: Is it possible to draw Fano plane in 2D-plane such that lines are straight lines and points of the Fano plane are drawn on the lines they belong to? (Are there 7 lines and 7 points in the plane such that there is a point in intersection of each two every two points are on a common line?)

14: Let (X, L) be a projective plane of order n. What are parameters of BIBD that corresponds to the incidence matrix of the projective plane?