MATH413 MIDTERM 2 - sample version

March 16 10:00-10:50am **Name: .....**

Answer as many problems as you can. Show your work. An answer with no explanation will receive no credit. Write your name on the top right corner of each page.

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6

1: By integrating the binomial expansion, prove that, for any integer n,

$$1+\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}+\cdots+\frac{1}{n+1}\binom{n}{n}=\frac{2^{n+1}-1}{n+1}$$

Name: .....

2: Determine the number of permutations of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ in which no even integer is in its natural position. For example, 43258176 is good, but 42358176 is bad because of the position of the 2.

Name: .....

3: Find the number of integers between 100 and 999 inclusive that are not divisible by 4, 6, or 9.

Name: .....

4: Determine the number of solutions of the equation $x_1+x_2+x_3+x_4+x_5 = 16$ in positive integers x_1, x_2, x_3, x_4 and x_5 not exceeding 7.

Name: .....

5: Prove that if $i < \lfloor n/2 \rfloor$ then $\binom{n}{i} \leq \binom{n}{i+1}$.

Name: .....

6: I have four exams to study for. I have 21 days to do it. I will study for any given exam during 10 days. For any given pair of exams, I will study them on the same day 5 times. For any given three exams, I will study them together on at most 3 of the days. Finally, I need to "relax" for 3 days (when I don't study at all). How many days do I have to spend studying for all four exams together?

Paper for attempts.