

**MATH413          MIDTERM 3 - sample version**

**April 27 10:00-10:50am**

**Name:** .....

Answer as many problems as you can. Best 5 out of 6 counts. Show your work. An answer with no explanation will receive no credit. Write your name on the top right corner of each page.

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6

**1:** Without using generating functions or iterations, determine  $h_n$ , where  $h_0 = 2$  and

$$\text{for every } n \geq 1 \quad h_n = 2h_{n-1} + 3^n.$$

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**2:** Determine the generating function for the sequence  $\{h_n\}_{n=0}^{\infty}$  that satisfies the relation  $h_n = -5h_{n-1} - 6h_{n-2} + 2^n$  for  $n \geq 2$  with initial conditions  $h_0 = h_1 = 0$ . Using the generating function find an explicit formula for  $h_n$  in this problem.

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**3:** Using exponential generating series, determine the number of ways to put 20 people in rooms  $A$  and  $B$  subject to the condition that room  $A$  has at least three people in it. (You need to actually extract the desired coefficients from your generating series.)

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4: Using the difference sequence method, give a close formula for

$$\sum_{k=1}^n k^3 + k^2 - 2k + 3.$$

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**5:** Prove that the number of partitions where no part appears more than two times equals the number of partitions where no part is a multiple of three. (*Hint: Write down the generating series for the former type of partition and manipulate the generating series.*)

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**6:** Three shorter questions:

- (a) Write the main recurrence relation for Catalan numbers and an explicit formula for them.
- (b) Give the definition of Stirling numbers of the second kind and calculate  $S(5, 3)$ .
- (c) List all self-conjugate partitions of the number 8.