CHANGE IN OBJECTIVE FUNCTION:

$$\Delta z = c_B^T \Delta x_B = c_B^T B^{-1} \Delta b = y^T \Delta b$$

SO IF b CHANGED BY Δb, OBJECTIVE FUNCTION CHANGED BY $y^T \Delta b$

- y^* GIVES SENSITIVITY OF THE SOLUTION

COMPLEMENTARY SLACKNESS

$$(P)\begin{cases} \min c^T x \\ Ax = b \\ x \geq 0 \end{cases} \qquad (D)\begin{cases} \max b^T y \\ A^T y \leq c \end{cases}$$

THEOREM:

LET x AND y BE FEASIBLE FOR (P) & (D)

x AND y ARE OPTIMAL PAIR IFF

a) $x_i > 0 \Rightarrow y^T a_i = c$

b) $x_i = 0 \Leftarrow y^T a_i < c$

PROOF:

$$a, b \Rightarrow (y^T A - c^T)\, x = 0$$

$$y^T A x - c^T x = 0$$

$$y^T b = c^T x \Rightarrow \text{OPTIMAL}$$

DUALITY

BACK: $\underbrace{(y^T A - c^T)}_{\leq 0} \underset{\geq 0}{x} = 0$

$\Rightarrow$ u_1 & b_1 MUST HOLD

B

FOR

$\min c^T x$ MAX $b^T y$

$Ax \geq b$ $A^T y \leq c$

$x \geq 0$ $y \geq 0$

SLACKNESS

EX

SUPPOSE DIET PROBLEM

$a_j x > b_j \Rightarrow y_j = 0$

$\hookrightarrow$ IN OPTIMAL SOLUTION WE GET $>$ THAN WHAT WE NEED OF b_j. $\Rightarrow$

$\Rightarrow$ b_j IS NOT „LIMITTING" IT IS „FOR FREE"

$\Rightarrow$ COST $y_j = 0$

SIMPLEX NOT P-TIME [KLEE - MINTY]

WE DON'T KNOW SIMPLEX METHOD, SO JUST A QUICK „IDEA"

GREEDY WAY - USE WITH HIGHEST COEFF

$$\min \quad 100x_1 + 10x_2 + x_3$$

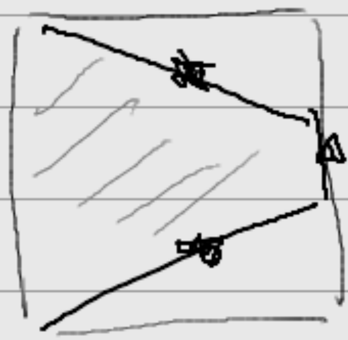
x_1	≤ 1	I
$20x_1 + x_2$	≤ 100	II
$200x_1 + 20x_2 + x_3$	≤ 10000	III

$\underset{IV}{x_1 \geq 0}$ $\underset{V}{x_2 \geq 0}$ $\underset{VI}{x_3 \geq 0}$

0)	$x_1 = 0$	$x_2 = 0$	$x_3 = 0$	0	IV V VI ... II
1)	$x_1 = 1$	$x_2 = 0$	$x_3 = 0$	100	I V VI
2)	$x_1 = 1$	$x_2 = 80$	$x_3 = 0$	900	I VI II
3)	$x_1 = 0$	$x_2 = 100$	$x_3 = 0$	1000	IV VI II
4)	$x_1 = 0$	$x_2 = 100$	$x_3 = 8000$	9000	IV II III
5)	$x_1 = 1$	$x_2 = 80$	$x_3 = 8200$	9100	I II III
6)	$x_1 = 1$	$x_2 = 0$	$x_3 = 9800$	9900	I V III
7)	$x_1 = 0$	$x_2 = 0$	$x_3 = 10000$	10000	IV V III

STATISFIED WITH EQUAL

IMAGE OF CUBE



ELLIPSOID METHOD [KHACHIYAN]

PROBLEM: H-POLYTOPE

$P = \{ x \in \mathbb{R}^n : a_j^T x \le b_j \}$

- FIND POINT IN P. LATER

(EQUIVALENT TO LIN. PROG)

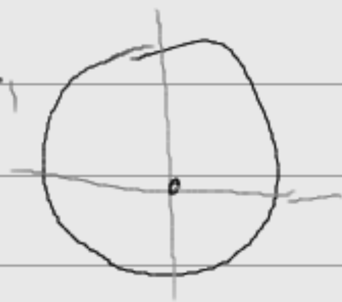
EXTRA ASSUMPTIONS:

- $\exists R \in \mathbb{R}$ $\quad P \subseteq B(0,R)$ HUGE
- $\exists r \in \mathbb{R}, \exists x \in P$ $\quad B(x,r) \subseteq P$

(R, r PART OF COMPLEXITY OF ALG.)

IDEA SKETCH: $B(0,R)$ = ELLIPSOID E_1

E_1



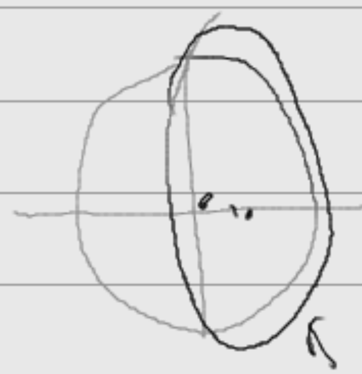
TEST IF 0 IS INSIDE

- YES - WIN
- NO ITERATE

P HERE

BINO

→ ROTATE



ELLIPSE E_2

USE LIN TRANSFORM → $\oplus$ AND REPEAT

E_{k+1} SMALLER THAN E_k

E_{k+1} IS ELLIPSOID OF SMALLEST VOLUME CONTAINING $\frac{1}{2} E_k$

$$\frac{VOL(E_{k+1})}{VOL(E_k)} \le e^{-\frac{1}{2(n+1)}} < 1$$

2n ITERATIONS ...

$$\frac{VOL(E_{k+2n})}{VOL(E_k)} < \frac{1}{2}$$

$$VOL\ E_1 = VOL(B(0,1)) \cdot R^n$$

$$VOL\ E_k \ge VOL(B(0,1)) \cdot r^n$$

$$R^n \cdot 2^{-k} \le r^n$$

$$2^{-k} \le 2^{n \log \frac{r}{R}}$$

$$k \ge n \log \frac{R}{r}$$

$\Rightarrow O(n^2 \log \frac{R}{r})$ ITERATIONS.

OPERATIONS

$\downarrow$

$O(n^4 \log \frac{R}{r})$

$O(n^2)$ FOR ITERATION

NOTES:

$\max c^T x$ $\qquad\qquad$ $\min y^T b$

s.t. $Ax \le b$ $\qquad\qquad$ s.t. $y^T A \ge c^T$

$x \ge 0$ $\qquad\qquad$ $y \ge 0$

→ FIND FEASIBLE POINT

$$c^T x \ge y^T b$$

$$Ax \le b \quad \Leftarrow \text{NOT FULL DIMENSION}$$

$$A^T y \ge c^T$$

$$x, y \ge 0$$

ADD + ε TO $\quad c^T x \ge y^T b - \varepsilon$

→ SOLVES UP TO ε _ GOOD ENOUGH

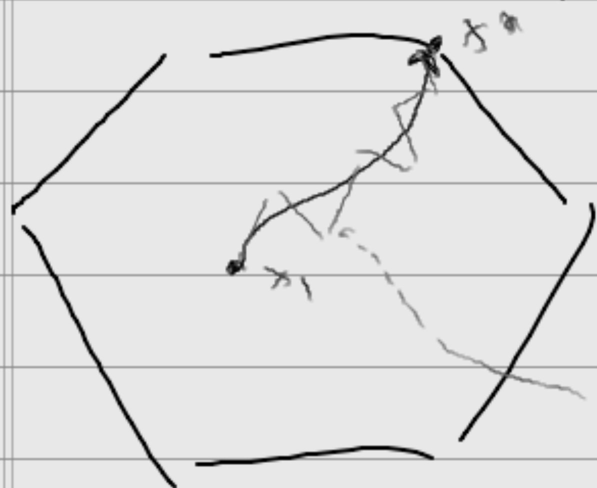
(ε MATTERS SMALL :D)

LUNCH × NATIONAL BUDGET

— NICE THEORY, DOES NOT WORK

FAST IN PRACTICE :C

INTERIOR POINT METHODS



START INSIDE WITH x_1
MOVE TOWARDS
OPTIMUM FROM INSIDE

MOVE „ALONG" PATH TO x^*

OPTIMIZE BY SMALL STEPS „LOCALLY"