$$(1-\lambda) f_0(x^*) \le (1-\lambda) f(y)$$

$\Rightarrow x^*$ IS GLOBAL MINIMIZER.
(BOTH CONVEXITY OF f_0 & FEAS. SET USED)

$\square$

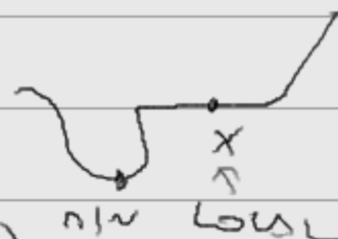
QUASICONVEX OPTIMIZATION

$$\left.\begin{aligned} \min\ & f_0(x)\\ \text{s.t.}\ & f_i(x)\le 0\\ & Ax = 0\end{aligned}\right\}\ (QC)$$

f_i .... CONVEX !

f_0 —— QUASICONVEX

LOCAL OPT. IS NOT GLOBAL



ALGORITHM: (USING CONVEX FEASIB.)

- KNOW $l \le f_0(x^*) \le u$ ← BOUNDS
- SOLVE

$$\left.\begin{aligned}\phi_{\frac{l+u}{2}}(x)&\le 0\\ f_i&\le 0\\ Ax&=0\end{aligned}\right\}\ \text{CONVEX}$$

BISECTION METHOD

UPDATE l OR u AND REPEAT

SPECIAL CONVEX PROGRAMS

- LINEAR PROGRAM

$$\begin{aligned} \min \quad & c^T x \\ \text{st} \quad & Gx \le h \\ & Ax = b \end{aligned}$$

- SURELY CONVEX :-)

- QUADRATIC PROGRAM

$$\begin{aligned} \min \quad & (1/2)\, x^T P x + q^T x + r \\ \text{st} \quad & Gx \le h \\ & Ax = b \end{aligned}$$

WHERE P POSITIVE SEMIDEFINITE

- QUADRATICALLY CONSTRAINED QUADRATIC PROGRAM (QCQP)

$$\begin{aligned} \min \quad & (1/2)\, x^T P_0 x + q_0^T x + r_0 \\ \text{st.} \quad & (1/2)\, x^T P_i x + q_i^T x + r_i \le 0 \quad i = 1 \ldots m \\ & Ax = b \end{aligned}$$

P_0 .. POS. SEMIDEF

P_i . POSITIVE SEMIDEF ELLIPSOIDS

(OTHERWISE NP-HARD)

• EXAMPLE: LEAST SQUARES

SOLVE $Ax = b$

$A \in \mathbb{R}^{m \times n}$ $m > n$

→ NO SOLUTION FIND „ALMOST SOLUTION"

Ax CLOSE TO b $\|Ax - b\|_2$ IS MINIMIZED

$\min \|Ax - b\|_2^2$

$$\|$$

$\min x^T A^T A x - 2 b^T A x + b^T b$

→ UNCONSTRAINED QUADRATIC PROGRAM

HAS SOLUTION $x = A^+ b$, $A^+ = (A^T A)^{-1} A^T$

↑ GENERALIZED INVERSE

CAN DO $\min \|Ax - b\|_2^2$

$\text{S.T. } l_i \leq x_i \leq h_i$ – BOUNDS ON x_i

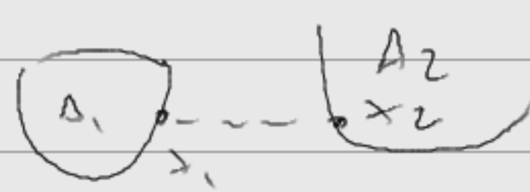
• EXAMPLE: DISTANCE BETWEEN POLYHEDRA

$\min \|x_1 - x_2\|_2^2$

$\text{S.T. } A_1 x_1 \leq b_1$

$A_2 x_2 \leq b_2$

FINDS



• SECOND-ORDER CONE PROGRAMMING (SOCP)

DEF NORM CONE

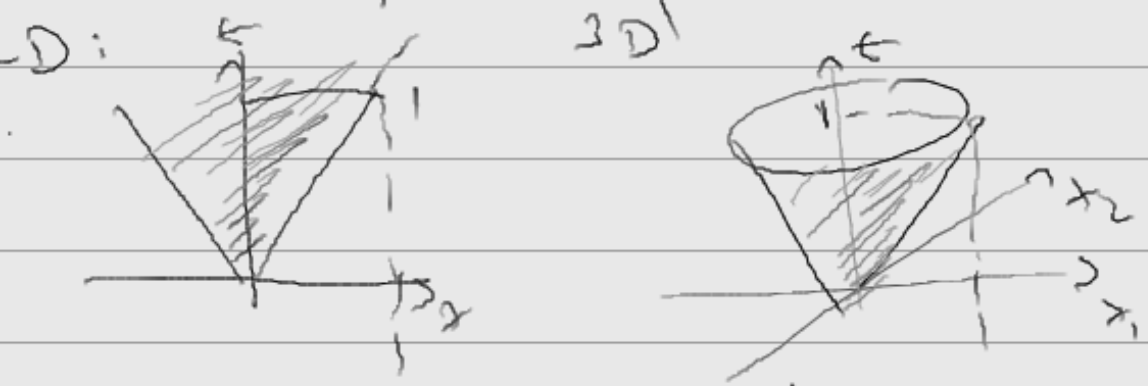
$$C = \{ (x, t) \mid \|x\| \leq t \} \subseteq \mathbb{R}^{n+1}$$

$x \in \mathbb{R}^n$ ↖ ANY NORM

DEF SECOND ORDER CONE $\|.\|_2$

$$C = \left\{ (x, t) \;\middle|\; (x, t) \begin{pmatrix} I & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \leq 0,\ t \geq 0 \right\}$$

2D: 3D



ALSO QUADRATIC CONE, LORENTZ CONE

ICE-CREAM CONE

(SOCP)
$$\begin{cases} \min\ f^T x \\ \text{s.t. } \|A_i x + b_i\|_2 \leq c_i^T x + d_i \quad i = 1 \dots m \\ Fx = g \end{cases}$$

$x \in \mathbb{R}^n$, $f \in \mathbb{R}^n$, $A_i \in \mathbb{R}^{n_i \times n}$ $b_i \in \mathbb{R}^{n_i}$

$F \in \mathbb{R}^{p \times n}$ $g \in \mathbb{R}^p$ $c_i \in \mathbb{R}^n$ $d_i \in \mathbb{R}$

NOTE: IF $u_i = 0\ \forall i$ THEN IT IS (LP)

NOTE: SHOULD GENERALIZE (QCQP)

- CONSTRAINTS EASY
- HOW TO FIX OBJECTIVE FUNCTION

(ALL QUASICONVEX?)

STANDART TRICK

EXAMPLE LP WITH UNCERTAIN COEF.

- ROBUST LINEAR PROGRAMMING

$\min\ c^T x$

S.T. $a_i^T x \le b_i$ SAY a_i IS NOT CLEAR

- DETERMINISTIC MODEL

S.T $a_i^T x \le b_i \quad \forall a_i \in A_i$

SOME RANGE

SAY $A_i = \{ \bar{a}_i + P_i u \mid \|u\|_2 \le 1 \}$

CENTER UNCERTAINTY

CONSTRAINTS:

$$\sup \{ a_i^T x \mid a_i \in A_i \} \le b_i$$

$\sup \{ a_i^T x \mid a_i \in A_i \} = \bar{a}_i^T x + \sup \{ u^T P_i^T x \mid \|u\|_2 \le 1 \}$

$= \bar{a}_i^T x + \| P_i^T x \|_2$ — CAUCHY-SCHWARTZ $|u^T P^T x| \le \|u\| \cdot \|P_i^T x\|$

NOW

$$\min \; c^T x$$
$$\text{S.T.} \quad \bar{a}_i^T x + \|P_i^T x\|_2 \le b_i \quad i = 1 \dots m$$

EXAMPLE 2 – a_i ARE RANDOM

FIND X ST. FOR RANDOM a_i

$$\min \; c^T x$$
$$\text{ST.} \quad P[a_i^T x \le b_i] \ge \eta_i$$

IF a_i NICE CAN BE SOLVED.

NO FURTHER DETAILS

- GEOMETRIC PROGRAMMING

MONOMIAL FUNCTION

$$f(x) = c x_1^{\alpha_1} x_2^{\alpha_2} \cdots x_n^{\alpha_n}$$

WHERE $c > 0$, $\alpha_i \in \mathbb{R}$, $x > 0$

POSYNOMIAL FUNCTION

$$f(x) = \sum_{k=1}^{K} c_k x_1^{\alpha_{1k}} x_2^{\alpha_{2k}} \cdots x_n^{\alpha_{kn}}$$

SUM OF MONOMIALS

$$(GP) \begin{cases} \min & f_0(x) \\ \text{s.t.} & f_i(x) \le 1 \\ & h_i(x) = 1 \end{cases}$$

$f_i(x)$ POSYNOMIAL, $h_i(x)$ MONOMIAL