11.3 BARRIER METHOD (PATH-FOLLOWING METHOD)

SOLVING LARGE t DIFFICULT SO:

BARRIER METHOD ALGORITHM

INPUT x STRICTLY FEASIBLE (SLATER PT)

$t > 0$

$\mu > 0$ INCREASE OF t

$\varepsilon > 0$ TOLERANCE

CENTERING:

$x^*(t)$ MINIMIZER OF $t f(x) + \phi(x)$

→ ITERATIVE METHOD WITH INITIAL POINT x

UPDATE: $x := x^*(t)$

STOP? : IF $m/t < \varepsilon$ STOP

$t := \mu t$

OUTER ITERATION : ONCE CYCLE

INNER ITERATION .. CYCLE INSIDE

CENTERING STEP - USE NEWTON METHOD

NOTES;

MUST BE THE INNER LOOP EXACT?

- NO, BUT THEN MAYBE NONDUAL SOLUTION. BUT STILL CONVERGES
- BUT FROM CLOSE SOL. TO "EXACT", NEWTON IS FAST

HOW TO PICK μ?

LARGE μ - FEW OUTER ITERATIONS

SMALL μ - FEW INNER ITERATIONS

(TYPICAL $\mu = 10 - 20$ WITH NEWTON)

HOW TO PICK t FOR INPUT?

LARGE t - DIFFICULT FIRST STEP

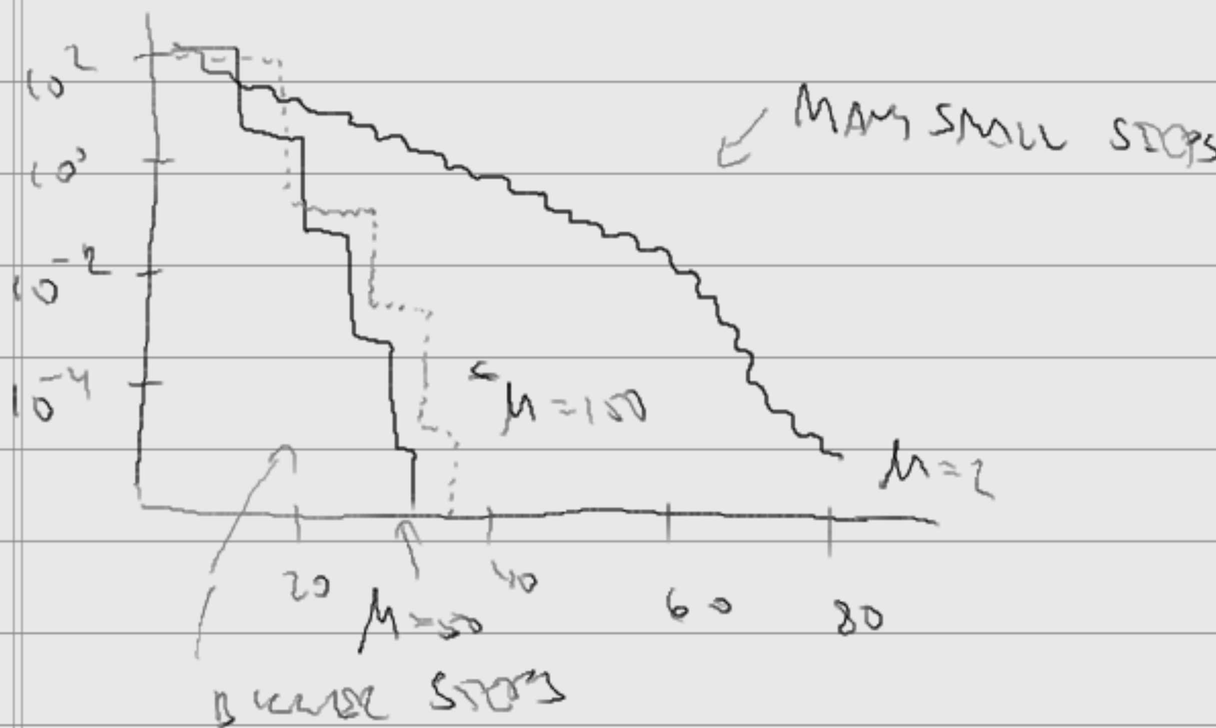
SMALL t - MANY OUTER ITERATIONS

· SET t ST $m/t \approx f(x) - f(x^*)$ IF KNOWN

OR IF λ DUAL FEASIBLE

USE $t = m/(f(x) - h(\lambda))$

EXAMPLE:



DETAILS IN BOOK IN SECTION 11.3.2

CONVERGENCE ANALYSIS:

OF OUTER ITERATIONS:

GAP: $$\frac{m}{\mu^k t} < \varepsilon \rightsquigarrow \left\lceil \frac{\log\left(\frac{m}{t\varepsilon}\right)}{\log \mu} \right\rceil + 1 = K$$

THE FIRST ITER.

INNER ITERATIONS BASED ON NEWTON'S METHOD

NEWTON STEP FOR KKT

1 CENTERING STEP: $[Hf(x_k)](x_{k+1}-x_k) = -\nabla f(x_k)$

$(t\,Hf(x_k) + H\phi(x_k))(x_{k+1}-x_k) = -t\nabla f(x_k) - \nabla\phi(x)$

SOLVING KKT:

$$\nabla f(x) + \sum \lambda_i g_i(x) = 0$$
$$-\lambda_i g_i(x) = 1/t$$
$$\lambda \geq 0$$

$\rightarrow$ REPLACE $\lambda_i = \dfrac{1}{-t g_i(x)}$

$f(x_k + v) \simeq$

$\simeq f(x_k) + v\nabla f(x_k)$

$$\nabla f(x) + \sum \frac{1}{-t g_i(x)} \nabla g_i(x) = 0$$

IN NEWTON METHOD FROM x_k USE APPROX

$$\nabla f(x_k + v) + \sum_{i=1}^{m} \frac{1}{-t g_i(x+v)} \nabla g_i(x+v) \simeq$$

$$\simeq \underbrace{\nabla f(x_k) + \sum \frac{1}{-t g_i(x)} \nabla g_i(x)}_{=g} + \underbrace{Hf(x_k)v + \sum \frac{1}{-t g_i(x_k)} Hg_i(x_k)v + \sum \frac{1}{t g_i(x)} \nabla g_i(x) \nabla g_i(x)^T v}_{Hv} = 0$$

$Hv = -g$

$\overset{(2)}{\odot}$ $H = Hf(x_k) + \frac{1}{t} H\phi(x_k)$

NOTE:

$$\nabla\phi(x) = \sum \frac{1}{-g_i(x)} \nabla g_i(x)$$

$$H\phi(x) = \sum \frac{1}{g_i(x)^2} \nabla g_i(x) \nabla g_i(x)^T +$$

$\overset{"}{\nabla^2}\phi(x)$

$$+ \sum \frac{1}{-g_i(x)} H g_i(x)$$

$$\left(Hf(x_k) + \frac{1}{t} H\phi(x_k)\right) v = \nabla f(x_k) + \frac{1}{t}\nabla\phi(x)$$

IF $v = (x_{k+1} - x_k)$, GIVES

EXACTLY THE SAME THING

$\Rightarrow$ 1 ITERATION OF CENTERING STEP IS SAME AS 1 ITERATION OF NEWTONS METHOD ON KKT.

11.4 FEASIBILITY & PHASE I METHODS

SLATER POINT REQUIRED FOR INTERPOINT.

PHASE I - FIND INTERIOR POINT x

PHASE II - USE x AS STARTING POINT

BASIC METHOD:

(V1)

$\min_{x,s} \; s \qquad s \in \mathbb{R}$

S.T. $g_i(x) \leq s$

IF SOLUTION (x^*, s^*) WHERE $s^* < 0$,

x^* IS STRICTLY FEASIBLE.

$s^* > 0$,

NOT FEASIBLE AT ALL.

$s^* = 0$,

FEASIBLE BUT NOT STRICTLY

(V2) MIN $1^T s$ → THAN (V1)

S.T. $g_i(x) \leq s_i$

$s_i \geq 0$

→ FOR INFEASIBLE PROBLEM SATISFY MORE

BY ADDING TO (V)

MIN s

S.T. $g_i(x) \le s$ ∴ $g_i(x) - s \le 0$

$f(x) \le M$ $\quad f(x) - M \le 0$

WHERE $M > f(x^0)$

$\Rightarrow$ CENTRAL PATHS OF AND (P) INTERSECTS

MIN $ts + \phi(x, s)$

CHARACTERIZATION:

$$\left[t f(x^0(t)) + \sum \frac{1}{-g_i(x^0(t))} \nabla g_i(x^0(t)) \right]$$

$$x: \quad \frac{1}{M - f_0(x)} \nabla f(x) + \sum \frac{1}{s - g_i(x)} \nabla g_i(x) = 0$$

$$s: \quad t - \sum_{i=1}^{m} \frac{1}{s - g_i(x)} = 0$$

FOR $s = 0$ AND $t = \dfrac{1}{M - f_0(x)}$ WE GET POINT ON CENTRAL PATH FOR (P).