11.7 PRIMAL-DUAL INTERIOR POINT METHODS

- APPLY NEWTON ON MODIFIED KKT CONDITIONS
- DOES NOT GENERATE NECESSARILY FEASIBLE POINTS (IN LIMIT)
- USUALLY MORE EFFICIENT THAN BARRIER METHODS

$$(P)\begin{cases} \min & f(x) \\ \text{ST} & g_i(x) \le 0 \end{cases}$$

KKT: $\exists \lambda$ ST.

1) $\lambda \ge 0$

2) $\nabla f(x) + \sum \lambda_i \nabla g_i(x) = 0$

3) $-\lambda_i g_i(x) = \frac{1}{t}$

$\Leftarrow x, \lambda$ FEASIBLE $\Rightarrow$ $\Rightarrow$ GAP m/t

$r_{DUAL} = \nabla f(x) + \sum \lambda_i \nabla g_i(x)$ ... DUAL RESIDUAL

$(r_{CENT})_i = -\lambda_i g_i(x) - 1/t$ ... CENTRALITY RESIDUAL

$r_t(x, \lambda) = (r_{DUAL}, r_{CENT})$

IDEA: USE NEWTON TO SOLVE

$r_t(x, \lambda) = 0$

LET $y = (x, \lambda)$ Δy -- STEP IN y

APROXIMATION:

$$r_t(y + \Delta y) \approx r_t(y) + \nabla r_t(y) \Delta y$$

SOLVE:

$$\left(Hf(x) + \sum \lambda_i H g_i(x) \right) \Delta x + \sum \nabla g_i(x) \cdot \Delta \lambda = R_{dual}$$

$$\left(-\lambda_i \nabla g_i(x)^T \Delta x - g_i(x) \cdot \Delta \lambda_i \right) = -(R_{cent})_i$$

SOLVE: $\Delta \lambda_i = g_i(x)^{-1} (R_{cent_i}) - g_i(x)^{-1} \lambda_i \nabla g_i(x) \Delta x$
& PLUG

$$\Rightarrow \quad H_{pd} \Delta x = -\nabla f(x) + \frac{1}{t} \sum \frac{1}{-g_i(x)} \nabla g_i(x)$$

$$H_{pd} = Hf(x) + \sum \lambda_i H g_i(x) + \sum \frac{\lambda_i}{-g_i(x)} \nabla g_i(x) \nabla g_i(x)^T$$

FOR BARRIER METHOD:

$$H_{BAR}\,\Delta X = -t\nabla f(x) + \sum \frac{1}{-g_i(x)} \nabla g_i(x) \nabla g_i(x)^T$$

DIVIDE BY t:

$$\frac{1}{t} H_{BAR} = Hf(x) + \sum \frac{1}{-t g_i(x)} H g_i(x) + \sum \frac{1}{t g_i(x)^2} \nabla g_i(x) \nabla g_i(x)^T$$

IF $-g_i(x)\lambda_i = \frac{1}{t}$ THEN $\frac{1}{t} H_{BAR} = H_{PD}$

$$\left(\sum -g_i(x)\lambda_i = \frac{m}{t} \right)$$

IF EXACT, JUST μ

ALGORITHM:

INPUT x S.T. $g_i(x) < 0$, $\lambda > 0$, $\varepsilon > 0$

ε FEASIBLE, $\mu > 1$, $y = (x, \lambda)$

PICK t:

$t := \mu m / \sum -g_i(x)\lambda_i$

COMPUTE Δy .. DIRECTION

COMPUTE $S\ (\leq 1)$ S.T.

$y = y + S\Delta y$

AND STILL $g_i(x) < 0$ & $\lambda > 0$

(STOP IF DUAL $\leq \varepsilon_{FEAS}$ & $\sum -g_i(x)\lambda \leq \varepsilon$)