MATH413 HW 3

due **Feb 13** before class, answer without justification will receive 0 points.

1: *(P. 68, #62)* Suppose a poker hand contains seven cards rather than five. Compute the probabilities of the following poker hands:

(a) a seven-card straight
(b) four cards of one rank and three of a different rank
(c) three cards of one rank and two cards of each of two different ranks
(d) two cards of each of three different ranks, and a card of a fourth rank
(e) there cards of one rank and four cards of each of four different ranks
(f) seven cards each of different ranks

2: *(P. 83, #5)* Show that if $n+1$ integers are chosen from the set $\{1, 2, \ldots, 3n\}$, then there are alway two which differ by at most 2.

3: *(P. 83, #8)* Use the pigeonhole principle to prove that the decimal expansion of rational number m/n eventually is repeating. For example,

$$\frac{34,478}{99,900} = 0.345125125125\cdots.$$

4: *(P. 83, #11.)* A student has 37 days to prepare for an examination. From past experience she knows that she will require no more than 60 hours of study. She also wishes to study at least 1 hour per day. Show that no matter how she schedules her study time (a whole number of hours per day, however), there is a succession of days during which she will have studied exactly 13 hours.

5: *(P. 83, #12.)* Read and try to understand Chinese remainder theorem. Show by example that the conclusion of the Chinese remainder theorem (Application 6) need not hold when m and n are not relatively prime.

6: *(P. 84, #15)* Prove that, for any $n+1$ integers $a_1, a_2, \ldots, a_{n+1}$, there exist two of the integers a_i and a_j with $i \neq j$ such that $a_i - a_j$ is divisible by n.

7: *(P. 84, #19)* (a) Prove that of any five points chosen within a equilateral triangle of side length 1, there are two whose distance apart is at most $\frac{1}{2}$.

(c) Determine an integer m_n such that if m_n points are chosen within an equilateral triangle of side length 1, there are two whose distance apart is at most $1/n$.

8: *(P. 85, #23)* The line segments joining 10 points are arbitrarily colored read or blue. Prove that there must exist three points such that the three line segments joining them are all red, or four points such that the six line segments joining them are all blue (that is, $r(3,4) \leq 10$).