FARKAS LEMMA

LET $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$ THEN EXACTLY ONE OF THE FOLLOWING HAPPENS:

a) $\exists\, x \in \mathbb{R}^n$ S.T. $Ax = b$ AND $x \geq 0$

b) $\exists\, y \in \mathbb{R}^m$ S.T. $y^T A \geq \vec{0}^T$ AND $y^T b < 0$

DEF

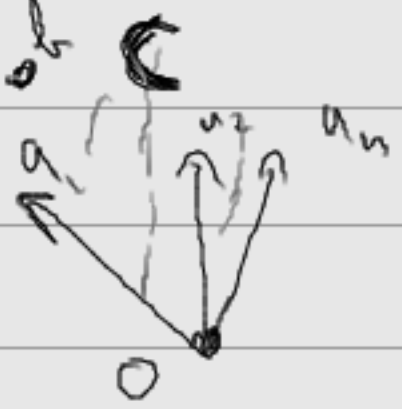
$a_1 \dots a_n \in \mathbb{R}^n$, CONVEX CONE (CONVEX, CLOSED) GENERATED BY $a_1 \dots a_n$:

$$C = \{ t_1 a_1 + t_2 a_2 + \dots + t_n a_n,\ t_i \geq 0 \}$$

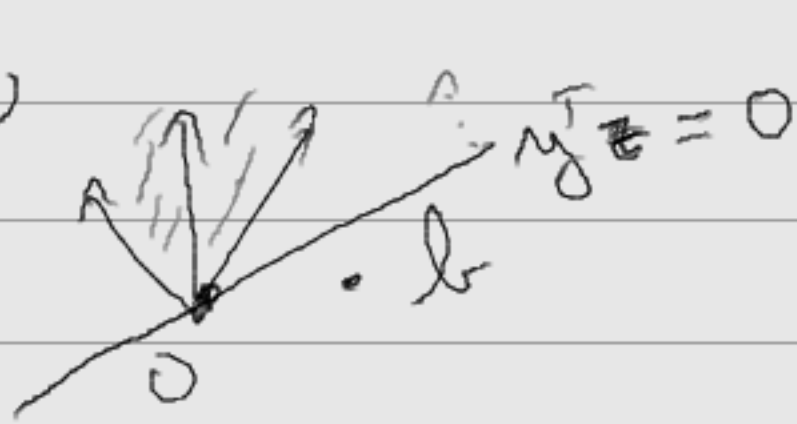
FARKAS GEOMETRICALLY

$A = (a_1 | a_2 | \dots | a_n)$

a) $b \in C$



b)



SEPARATION THEOREM: IF $Ax = b$, $x \geq 0$ HAS NO SOLUTION $\Rightarrow b \notin C$. $\overset{\text{S.T.}}{\Rightarrow}$

$\exists\, y \in \mathbb{R}^m$, $\alpha \in \mathbb{R}$: ($q = 0$??)

$$y^T q \geq \alpha > y^T b \quad \forall q \in C. \quad \square$$