Math-484 Homework #1 (warm up)

I will finish the homework before 11am Sep 4. If I spot a mathematical mistake I will let the lecturer know as soon as possible.
I will write clearly and neatly as the grader is not an expert in cryptography. I will sign each paper of my work and indicate if I am C14 (4 hours student).

1: (*I will check if I can find minimizers and maximizers of smooth functions*)
Find the local and global minimizers and maximizers of the following functions:
(a) $f(x) = x^2 + 2x$
(b) $f(x) = x^2 e^{-x^2}$
Verify answers using `http://www.wolframalpha.com` and include printout of the solution.
Hint: Use first and second derivatives of $f(x)$ and critical points.

2: (*I will recall few basic definitions*)
Determine the dimension of the smallest subspace of $\mathbb{R}^4$ that contains vectors $(0, 1, 0, 1)$, $(3, 4, 1, 2)$, $(6, 4, 2, 0)$ and $(-3, 1, -1, 3)$.

3: (*I will recall what are determinants*)
Compute determinants of the following real matrices:

(a) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & -2 & 1 & 0 \\ 4 & a & b & 1 \\ 1 & c & d & 4 \\ 0 & 1 & -2 & 0 \end{pmatrix}$ where $a, b, c, d \in \mathbb{R}$ are parameters

Verify answer (b) using `http://www.wolframalpha.com` and include printout of the solution.

4: (*I will recall what are eigenvalues and eigenvectors*)
Compute eigenvalues and eigenvectors of the following real matrix

$$A = \begin{pmatrix} 2 & 6 \\ 6 & -3 \end{pmatrix}$$

5: (*I will recall how to compute angle of two vectors*)
Compute the angle between vectors $(1, 1, 0)$ and $(2, 2, \sqrt{2})$.

6: (*I will check the definition of semidefinity and recall computing with matrices and vectors.*)
Suppose that A is a square matrix and suppose that there is another matrix B such that

$A = B^T B$. Show that A is positive semidefinite.

Hint: Recall that $\mathbf{y}^T B^T \mathbf{x} = (B\mathbf{y})^T \mathbf{x}$