Math-484 Homework #10 (penalty method)

I will finish the homework before 11am Nov 13. If I spot a mathematical mistake I will let the lecturer know as soon as possible.

I will write clearly and neatly as the grader is not an expert in cryptography. I will sign each paper of my work and indicate if I a 4 credits student.

Score counted for **3 credit - best 5 out of 6, 4 credit - 6 out of 6**

1: (*What is a dual of a linear program?*)
My grandma is preparing for the winter. Prices of vegetable usually go up and she wants to know if it is better to buy vegetables or vitamin pills. Help her determine when are prices of vitamin pills better than buying vegetable.

Consider the following diet problem, where you minimize the total cost and you want to get at least 100% of recommended daily intake of vitamins A,C, and K, which my grandma cares about. The following table lists for one cup of vegetable percentage or recommended daily intake of vitamins A,C,K and price in CZK (currency used by my grandma).

item	A	C	K	cost
carrot	407	12	20	1.8
spinach	377	30	1110	24.0
broccoli	11	135	115	2.3
cabbage	0	42	66	0.7

Write a linear program, solve it (with APMonitor) and give a list of what vegetable to buy to minimize the overall cost. Then write the dual of the program and solve the dual problem (using APMonitor). What is the interpretation of dual variables?

Local pharmacy has the following offer: 30 pills of vitamin A for 66CZK, 120 pills of vitamin C can be bought for 79CZK, and 120 pills of vitamin K for 315CZK.

One pill covers the recommended daily intake for one day of the corresponding vitamin. Is it better to buy vegetable or buy the pills? Is it worth to buy any of the vitamin pills in combination with vegetable (suppose it is possible to eat fractions of the pills)?

Use APMonitor to solve the programs and include printouts of the programs.

(The data in this question are real in the country where my grandma is living.)

2: (*Penalty functions warm-up*)
Consider the following program:

$$(P)\begin{cases}\text{Minimize} & f(x) = x^2 - 2x\\ \text{subject to} & 0 \le x \le 1.\end{cases}$$

(a) Sketch the graphs of the Absolute Value and Courant-Beltrami Penalty Terms for (P).
(b) For each positive integer k, compute minimizer x_k of the corresponding unconstrained objective function $P_k(x)$ with the Courant-Beltrami Penalty Term.
(c) For each positive integer k, compute the minimizer x_k of the corresponding unconstrained objective function $F_k(x)$ with the Absolute Value Penalty Term.

3: (*More penalty*)

a) Use the penalty function method with the Courant-Beltrami penalty term to solve the problem (P).

$$(P) \quad \begin{cases} \text{Minimize} & f(x_1, x_2) = x_1 + x_2 \\ \text{subject to} & x_1^2 - x_2 \leq 2 \end{cases}$$

b) Show that the objective function $F_k(\mathbf{x})$ corresponding to the Absolute value penalty term has no critical points off the parabola

$$x_1^2 - x_2 = 2$$

for $k > 1$ and compute the minimizer of $F_k(\mathbf{x})$.

4: (*Penalty again*)
Use the Penalty Function Method with Courant-Beltrami Penalty Term to minimize

$$f(x, y) = x^2 + y^2$$

subject to constraint $x + y \geq 1$.

5: (*What is* (P^ε)*?*)
Let $\varepsilon > 0$. Show that if a vector λ is a feasible for the dual (D) of a convex program (P), then λ is also feasible for (D^ε).

6: (*Is superconsistency necessary for no duality gap?*)
We know that if a convex program is superconsistent, then $MP = MD$. Show that the converse is not true. That is: find a convex program that is not sueprconsistent and yet $MP = MD$.