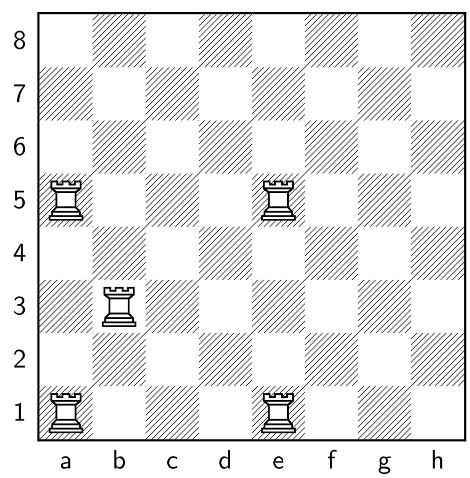
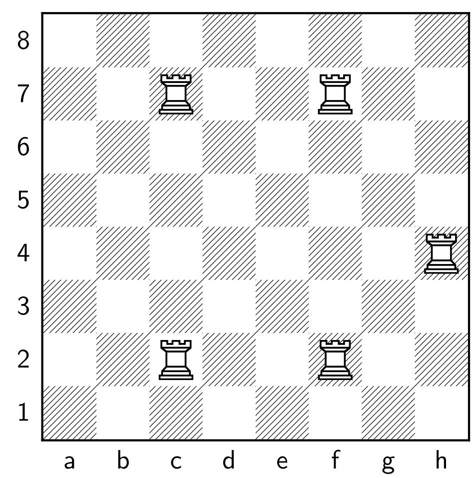
MATH413 HW 3

due **Feb 19** before class, answer without justification will receive 0 points.

1: *(P. 66, #50)* In how many was can five identical rooks be placed on the squares of an 8-by-8 board so that four of them form the corners of a rectangle with sides parallel to the sides of the board?
Examples of counted configurations:





2: *(P. 68, #64)* Let n be a positive integer. Suppose we choose a sequence $i_1, i_2, \ldots, i_n$ of integers between 1 and n at random.
(a) What is the probability that the sequence contains exactly $n-2$ different integers?

3: *(P. 83, #8)* Use the pigeonhole principle to prove that the decimal expansion of rational number m/n eventually is repeating. For example,

$$\frac{34,478}{99,900} = 0.345125125125\cdots.$$

4: *(P. 83, #11.)* A student has 37 days to prepare for an examination. From past experience she knows that she will require no more than 60 hours of study. She also wishes to study at least 1 hour per day. Show that no matter how she schedules her study time (a whole number of hours per day, however), there is a succession of days during which she will have studied exactly 13 hours.

5: *(P. 83, #12.)* Read and try to understand Chinese remainder theorem. Show by example that the conclusion of the Chinese remainder theorem (Application 6) need not hold when m and n are not relatively prime.

6: *(P. 84, #15)* Prove that, for any $n+1$ integers $a_1, a_2, \ldots, a_{n+1}$, there exist two of the integers a_i and a_j with $i \neq j$ such that $a_i - a_j$ is divisible by n.

7: *(P. 84, #19)* (a) Prove that of any five points chosen within a equilateral triangle of side length 1, there are two whose distance apart is at most $\frac{1}{2}$.

(c) Determine an integer m_n such that if m_n points are chosen within an equilateral triangle of side length 1, there are two whose distance apart is at most $1/n$.