MATH413 HW 4

due **Mar 5** before class, answer without justification will receive **0 points**.
Staple all your papers.

1: The line segments joining 9 points are arbitrarily colored read or blue. Prove that there must exist three points such that the three line segments joining them are all red, or four points such that the six line segments joining them are all blue (that is, $r(3,4) \leq 9$).

2: *(P. 85, #24)* Let x and t be positive integers with $x \geq t$. Determine the Ramsey number $r_t(t,t,x)$.
(See page 82 for definition of r_t.)

3: *(P. 155, #6 [similar])* What is the coefficient of x^6y^{13} in the full expansion of $(3x-4y)^{19}$? What is the coefficient of x^9y^9? (There is no misprint in this question!)

4: *(P. 155, #9)* Evaluate the sum

$$\sum_{k=0}^{n}(-1)^k\binom{n}{k}10^k.$$

5: *(P. 156, #13)* Find one binomial coefficient equal to the following expression:

$$\binom{n}{k}+3\binom{n}{k-1}+3\binom{n}{k-2}+\binom{n}{k-3}.$$

6: *(P. 156, #15)* Prove that for every integer $n > 1$,

$$\binom{n}{1}-2\binom{n}{2}+3\binom{n}{3}+\cdots(-1)^{n-1}n\binom{n}{n}=0.$$

7: *(P. 156, #20)* Find integers a,b, and c such that

$$m^3 = a\binom{m}{3}+b\binom{m}{2}+c\binom{m}{1}$$

for all m. Then sum the series $1^3+2^3+3^3+\cdots+n^3$.

8: *(P. 157, #27)* Let n and k be positive integers. Give a combinatorial proof of the identity

$$n(n+1)2^{n-2} = \sum_{k=1}^{n}k^2\binom{n}{k}$$

9: *(P. 157, #24)* Consider a three-dimensional grid whose dimensions are 10 by 15 by 20. You are at the front lower left corner of the grid and you wish to get to the back top right corner 45 steps away. How many different routes are there that take exactly 45 steps?

10: A chessboard 8×8 contains 33 rooks. Show that there exists 5 rooks that are in pairwise non-attacking position. Give an example of placement of 32 rook on 8×8 chess board that does not contain 5 rooks that are in pairwise non-attacking position.