MATH413 HW 5

due **Mar 12** before class, answer without justification will receive **0 points**. **Staple all your papers.**

1: *(P. 158, #36)* Prove

$$\sum_{k=0}^{n} \binom{m_1}{k} \binom{m_2}{n-k} = \binom{m_1+m_2}{n}$$

using binomial theorem and the relation $(1+x)^{m_1}(1+x)^{m_2} = (1+x)^{m_1+m_2}$.

2: *(P. 159, #40)* What is the coefficient of $x_1^3 x_2^3 x_3 x_4^2$ in the expansion of

$$(x_1 - x_2 + 2x_3 - 2x_4)^9?$$

3: *(P. 160, #46)* Use Newton's binomial theorem to approximate $\sqrt{30}$. *(Hint: See page 148 and 149. First three digits after the decimal point is enough.)*

4: If you pick an integer between 1 and 1000 (including 1 and 1000), what is the probability that it is either divisible by 7 or 5 or even (or two or all of these)?

5: How many multisets of 3 letters can be formed from letters M,I,S,S,I,S,S,I,P,P,I?

6: Count the number of integer solutions of

$$x_1 + x_2 + x_3 + x_4 = 28,$$

where $0 \le x_1 \le 6$, $0 \le x_2 \le 10$, $0 \le x_3 \le 15$, $0 \le x_4 \le 21$.

7: How many ways are there to distribute k distinct objects into five (distinct) boxes with at least one empty box?

8: Count the number of placements of 8 tokens on 4×4 board such that there exists a row or a column containing 4 tokens.