MATH413 HW 6

due **April 2** before class, answer without justification will receive 0 points. Staple all your papers.

1: *(P. 199, #12)* Determine the number of permutations of $\{1, 2, \ldots, 8\}$ with exactly four integers in its natural position.

2: Prove that D_n is an odd number if and only if n is an even number.

3: What is the number of ways to place six nonattacking rooks on the 6-by-6 boards without forbidden positions $\times$ as shown?

				$\times$	$\times$
					$\times$
		$\times$			
		$\times$	$\times$		
$\times$	$\times$				
$\times$	$\times$				

4: *(P. 201, #28)* A carousel has eight seats, each representing a different animal. Eight boys are seated on the carousel but facing inward, so that each boy faces another (each boy looks at another boy's front). In how many ways can be the boys change seats so that each faces a different boy? How does the problem change if all seats are identical?

5: *(P. 201, #30)* How many circular permutations are there of the multiset

$$\{3 \cdot a, 4 \cdot b, 2 \cdot c, 1 \cdot d\},$$

where, for each type of letter, all letters of that type do not appear consecutively?

6: When I walk up the stairs I can either go on the next stair or I can skip one (if I want to have more exercise or if I am late for class). In how many ways I can climb up n stairs?

7: Let $f_0, f_1, f_2, \cdots, f_n, \cdots$ denote the Fibonacci sequence. Prove the following identities:
(a) $f_n^2 - f_{n+1}f_{n-1} = (-1)^{n-1}$ for $n \geq 1$.
(b) $f_{n-1}^2 + f_n^2 = f_{2n-1}$ and $f_{n-1}f_n + f_nf_{n+1} = f_{2n}$.

8: Let $f_0, f_1, f_2, \cdots, f_n, \cdots$ denote the Fibonacci sequence. By evaluating each of the following expressions for small values of n, conjecture a general formula and then prove it, using mathematical induction and the Fibonacci recurrence:
(a) $f_0 + f_2 + f_4 + \cdots + f_{2n}$

(b) $f_0 - f_1 + f_2 - f_3 + \cdots + (-1)^n f_n$

9: *(P. 258, #4)* Prove that the Fibonacci sequence is the solution of the recurrence relation

$$a_n = 5a_{n-4} + 3a_{n-5}$$

for $n \geq 5$ and where $a_0 = 0$, $a_1 = 1$, $a_2 = 1$, $a_3 = 2$, and $a_4 = 3$. Then use this formula to show that the Fibonacci numbers satisfy the condition that f_n is divisible by 5 if and only if n is divisible by 5.

10: Find generating functions for the following sequences (and simplify them - i.e. without infinite sums):
(a) $0, 0, 0, 4, -4, 4, -4, 4, -4, 4, -4, \ldots$
(b) $1, 4, 9, 16, 25, 36, 49, \ldots$
Use derivative, integral and/or decomposing one sequence into several simpler ones.