

MATH413 HW 7

due **April 9** before class, answer without justification will receive 0 points. Staple all your papers.

1: (*P. 259, #14*) Let S be the multiset $\{\infty \cdot e_1, \infty \cdot e_2, \infty \cdot e_3, \infty \cdot e_4\}$. Determine the generating function for the sequence $h_0, h_1, h_2, \dots, h_n, \dots$, where h_n is the number of n -combinations of S with the following added restrictions:

- (a) Each e_i occurs an odd number of times.
- (b) Each e_i occurs a multiple-of-3 number of times.
- (c) The element e_1 does not occur, and e_2 occurs at most once.

2: (*P. 259, #17*) Determine the generating function for the number h_n of bags of fruit of apples, oranges, bananas, and pears in which there are an even number of apples, at most two oranges, a multiple of three number of bananas, and at most one pear. Then find a formula for h_n from the generating function.

3: (*P. 260, #19*) Let $h_0, h_1, h_2, \dots, h_n, \dots$ be the sequence defined by $h_n = \binom{n}{2}, (n \geq 0)$. Determine the generating function for the sequence.

4: (*P.260, #24*) Let S denote the multiset $\{\infty \cdot e_1, \infty \cdot e_2, \dots, \infty \cdot e_k\}$. Determine the exponential generating function for the sequence $h_0, h_1, h_2, \dots, h_n, \dots$, where $h_0 = 1$ and for $n \geq 1$,

- (a) h_n equals the number of n -permutations of S in which each object occurs an odd number of times.
- (d) h_n equals the number of n -permutations of S in which e_1 occurs at most once, e_2 occurs at most twice, ... , e_k occurs at most k times.

5: (*P.260, #27*) Determine the number of n -digit numbers with all digits odd, such that 1 and 3 each occur a nonzero, even number of times.

6: How many sequences of length n can be formed by letters A, B and C , where the number of A 's and B 's is odd?