You have to show your work and write down your proof.

1:   Solve following recurrence using characteristic equation.

$$h_n = 2h_{n-1} - 2^n \quad \text{for } n \geq 1, \quad \text{and } h_0 = 3.$$

Verify your answer for $n = 1$.
(Solution using generating functions or iterations will receive 0 points)

2: *Game.* Consider an ordered sequence of elements (tokens) numbered $1, 2, \ldots, n$ piled up in stack A with token 1 on the top and token n at the bottom, while stacks B and C are initially empty. The only type of legal move in this game is one where only the top token of a stack can be moved to the top of the stack to its right. The game ends when all tokens are on the stack C. Find the number of possible permutations of the tokens $1, 2, \ldots, n$ that can be created on stack C.

Example of initial state for $n = 3$.

1
2
3

A B C