MATH201 MIDTERM 3 - Practice problems

1: *11.2.9* Define a relation R on $\mathbb{Z}$ as xRy if and only if $4|(x+3y)$. Prove R is an equivalence relation. Describe its equivalence classes.

2: *11.4.6* Suppose $[a],[b] \in \mathbb{Z}_6$ and $[a]\cdot[b]=[0]$. Is it necessarily true that either $[a]=[0]$ or $[b]=[0]$?

3: *12.4.10* Consider the function $f:\mathbb{R}^2 \to \mathbb{R}^2$ defined by the formula $f(x,y)=(xy,x^3)$. Find a formula for $f\circ f$.

4: *12.5.6* The function $f:\mathbb{Z}\times\mathbb{Z}\to\mathbb{Z}\times\mathbb{Z}$ defined by the formula $f(m,n)=(5m+4n,4m+3n)$ is bijective. Find its inverse.

5: Let $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$. Assume $g\circ f$ is a surjective function. Is it true that f or g must be surjective?
(This question is: good - bad - ugly? Difficulty: 0-9:)

6: *13.3.2* Prove that the set $\mathbb{C}$ of complex numbers is uncountable.

7: *13.2.?* Prove or disprove: The set $\mathbb{Z}^{100}$ is countably infinite.

8: *1.3.?* Show reverse triangle inequality. That is for all $a,b\in\mathbb{R}$ we have

$$||a|-|b|| \le |a-b|.$$

9: *2.1.?* Show that if If $\{x_n\}$ is a convergent sequence, then $\{|x_n|\}$ is convergent and $\lim_{n\to\infty}|x_n| = |\lim_{n\to\infty}x_n|$
(This question is: good - bad - ugly? Difficulty: 0-9:)

10: *2.1.?* Let $r,s\in\mathbb{R}$. Prove that for every $\varepsilon>0$, $r+\varepsilon>s$ if and only if $r\ge s$.

11: *2.2.5* Let $x_n = n-\cos(n)$. Use the squeeze lemma to show that $\{x_n\}$ converges and find the limit.

12: *2.2.7* True or false, prove or find a counterexample. If $\{x_n\}$ is a sequence such that $\{x_n^2\}$ converges, then $\{x_n\}$ converges.

13: *2.2.8* Show that

$$\lim \frac{n^2}{2^n} = 0.$$

Hint: Ratio test.

14: *2.2.?* Let $q\in(0,1)$. Show that $\lim_{n\to\infty} n\cdot q^n = 0$.
Hint: Ratio test.

15: *2.3.2* Suppose $\{x_n\}$ is a bounded sequence. Define b_n as in Definition 2.3.1. Show that $\{b_n\}$ is an increasing sequence.

16: *2.3.7 (c)* Find bounded sequences $\{x_n\}$ and $\{y_n\}$ such that

$$(\liminf_{n\to\infty} x_n) + (\liminf_{n\to\infty} y_n) < \liminf_{n\to\infty}(x_n + y_n)$$

Hint: Look for examples that do not have a limit.

17: *2.4.1* Prove that $\{\frac{n^2-1}{n^2}\}$ is Cauchy using directly the definition of Cauchy sequences.

18: *2.4.1'* Prove that $\{\frac{n^2-1}{n^2}\}$ is convergent directly using the definition of a limit of a sequence.

19: *2.4.?* Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be two Cauchy sequences. Define $c_n = |a_n - b_n|$. Show that $\{c_n\}_{n=1}^{\infty}$ is a Cauchy sequence.