MATH 201 HW 8

due **Mar 11** before class.

Staple all your papers. Write carefully, unreadable answers will not receive any credit. Write your opinion about every question - good - bad - ugly - (or some other) and difficulty.

Please write your section or time of your class on you HW.

1: *(Induction in geometry)* Let us draw n lines in the plane in such a way that no two are parallel and no three intersect in a common point. Prove that the plane is divided into exactly

$$\frac{n(n+1)}{2}+1$$

parts by the lines.
(This question is: good - bad - ugly? Difficulty: 0-9:)

2: Prove by induction:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

(This question is: good - bad - ugly? Difficulty: 0-9:)

3: *Incorrect proof* Find a mistake in the *proof* of the following theorem.

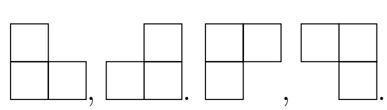
Theorem 1. *Every natural number $n \geq 2$ has a unique prime factorization.*

Incorrect proof by induction: Let $n \geq 2$ be a natural number. If n is a prime, then n is the unique prime factorization. If n is not a prime number, then there exist a and b natural numbers such that $n > a \geq 2$, $n > b \geq 2$ and $n = a \times b$. By induction, a has a unique prime factorization $a_1, \ldots, a_k$ and b has a unique prime factorization $b_1, \ldots, b_l$. Then $a_1, \ldots, a_k, b_1, \ldots, b_l$ is the unique prime factorization of n. □

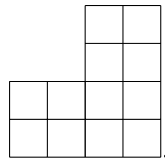
Note that the base case of the induction is present - if $n = 2$, then n is prime and the case is discussed in the proof. I mean - the mistake is something different than the basic step.
(This question is: good - bad - ugly? Difficulty: 0-9:)

4: For any integer $n \geq 0$, it follows $3|(5^{2n}-1)$.
(This question is: good - bad - ugly? Difficulty: 0-9:)

5: *Tiling* Use induction to show that for every natural number $n \geq 1$, it is possible to tile the grid $(1, \ldots, 2^n) \times (1, \ldots, 2^n)$, that is missing piece $(1, \ldots, 2^{n-1}) \times (1, \ldots, 2^{n-1})$ By pieces of L shape, that is



Example of the grid to tile for $n = 2$:



(This question is: good - bad - ugly? Difficulty: 0-9:   )

6: Let F_n is the nth Fibonacci number. Prove that

$$F_2 + F_4 + F_6 + F_8 + \cdots + F_{2n} = F_{2n+1} - 1$$

holds for all $n \geq 1$.
(This question is: good - bad - ugly? Difficulty: 0-9:   )

7: Show that the greatest common divisor of any two consecutive Fibonacci numbers is 1. In other words, for all $n \geq 1$, $\gcd(F_n, F_{n+1}) = 1$, where F_n is the nth Fibonacci number.
(This question is: good - bad - ugly? Difficulty: 0-9:   )

8: Let F_n is the nth Fibonacci number. Prove that

$$F_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}.$$

This may require some extra study.
(This question is: good - bad - ugly? Difficulty: 0-9:   )