**MATH 201** **HW 13 - Chapters 2.2, 2.3, 2.4, 2.5 from Basic analysis**

due **Apr 29** before class.

**Staple** all your papers. Write carefully, unreadable answers will not receive any credit. Write your opinion about every question - good - bad - ugly - (or some other) and difficulty.

Please write your section or time of your class on you HW.

**1:** *2.2.?* Let $\{x_n\}$ be a convergent sequence with limit 2. Is it possible to use the ratio test to determine that $\{x_n\}$ is convergent? Give a counterexample or a proof.
*(This question is: good - bad - ugly? Difficulty 0-9: )*

**2:** *2.2.?* Let $\{y_n\}$ is a convergent sequence such that $\lim y_n \neq 0$ and $y_n \neq 0$ for all $n \in \mathbb{N}$. Show that

$$\lim_{n\to\infty} \frac{1}{y_n} = \frac{1}{\lim_{n\to\infty} y_n}$$

**3:** *2.3.?* We know that a **bounded** sequence $\{x_n\}$ is convergent and converges to $x$ if and only if every convergent subsequence $\{x_{n_k}\}$ converges to $x$.
Is it true that a ~~**bounded**~~ sequence $\{x_n\}$ is convergent and converges to $x$ if and only if every convergent subsequence $\{x_{n_k}\}$ converges to $x$? (In other words, is the assumption on $\{x_n\}$ being bounded necessary?)

**4:** *2.3.8 (b)* Find bounded sequences $\{x_n\}$ and $\{y_n\}$ such that

$$(\limsup_{n\to\infty} x_n) + (\limsup_{n\to\infty} y_n) > \limsup_{n\to\infty}(x_n + y_n)$$

Hint: Look for examples that do not have a limit.

**5:** *2.4.?* Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be two Cauchy sequences. Define $c_n = |a_n - b_n|$. Show that $\{c_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

**6:** *2.5.?* Use the definition of convergent series to show that $\sum_{n=0}^{\infty} \frac{1}{3^n}$ is convergent.

**7:** *2.5.?* Show that $\sum_{n=0}^{\infty} \frac{1}{2^n}$ is a Cauchy series by verifying the definition when is series Cauchy.

**8:** *2.5.? (Linearity of series)* Let $\sum x_n$ and $\sum y_n$ be convergent series. Show that $\sum(x_n+y_n)$ is also convergent and

$$\left(\sum_{n=1}^{\infty} x_n\right) + \left(\sum_{n=1}^{\infty} y_n\right) = \sum_{n=1}^{\infty}(x_n + y_n).$$