CHAPTER 9 - DISPROOF

RECALL:

STATEMENTS
- TRUE THEOREMS
- FALSE
- ??? CONJECTURE

WHAT IF CONJECTURE IS TRUE?
PROOF

WHAT IF CONJECTURE IS FALSE?
DISPROOF

SAY CONJECTURE: P PROVE P PROOF
  PROVE $\neg P$ DISPROOF

9.1 DISPROVING UNIVERSAL STATEMENTS

CONJECTURE $\forall x \in S, P(x)$

DISPROOF: $\neg(\forall x \in S, P(x)) = \exists x \in S, \neg P(x)$

IT IS ENOUGH TO FIND ONE x S.T. $\neg P(x)$. ($P(x)$ IS NOT TRUE).
x IS CALLED COUNTEREXAMPLE

CONJECTURE: $P(x) \Rightarrow Q(x)$

DISPROOF: $\exists x,\ P(x) \ \& \ \neg Q(x)$

CONJECTURE

$\forall n \in \mathbb{Z},\ n^2 - n + 11$ IS PRIME

FIRST EXPERIMENT (FINDING CHEAP COUNTEREXAMPLE)

n	-2	-1	0	1	2	3	4	...	11
n^2-n+11	17	13	11	11	13	17	23		121

DISPROOF:

LET $n = 11$, THEN $11^2 - 11 + 11 = 11^2$, WHICH IS NOT A PRIME $\blacksquare$

JUST 1 EXAMPLE ENOUGH

- EXAMPLE GIVEN
- EXPLAINED WHY IT IS A COUNTEREXAMPLE

9.2 DISPROVING EXISTENCE STATEMENTS

CONJECTURE: $\exists x \in S, P(x)$

DISPROOF $\neg(\exists x \in S, P(x)) = \forall x \in S, \neg P(x)$

FINDING ONE x ST $\neg P(x)$ IS NOT ENOUGH! COUNTEREXAMPLE IS NOT RIGHT APPROACH.

CONJECTURE:

$\exists x \in \mathbb{R} : x^4 < x < x^2$

TRUE OR FALSE ???

FINDING x

- $x \neq 0$

$x < 0$ ... $0 < x^4$ => $\boxed{x > 0}$

$x^4 < x$ $x < x^2$

$x^3 < 1$ $1 < x$

$x < 1$ LOOKS WRONG

DISPROOF:

$\neg (\exists x \in \mathbb{R} : x^4 < x < x^2)$

$\forall x \in \mathbb{R} : \neg (x^4 < x < x^2)$

$\forall x \in \mathbb{R} \quad \neg ((x^4 < x) \,\&\, (x < x^2))$

CLAIM $\forall x \in \mathbb{R} \quad (x^4 > x) \vee (x > x^2)$

PROOF:

SUPPOSE FOR CONTRADICTION

$\exists x \in \mathbb{R},\ x^4 < x < x^2$

SINCE $x^4 \geq 0$, $x > 0$

$x^4 < x$ $\qquad$ $x < x^2$

$x^3 < 1$ $\qquad$ $1 < x$

$x < 1$ $\qquad$ $\parallel$

$x < x$ WHICH IS CONTRADICTION. $\square$

CONJECTURE P

DISPROOF: SUPPOSE P FOR CONTRADICTION...

... CONTRADICTION $\square$

9.3 DISPROOF BY CONTRADICTION

CONJECTURE: P

DISPROOF:

SUPPOSE P IS TRUE AND DEDUCE CONTRADICTION

EX $\exists x \in \mathbb{R}, x^4 < x < x^2$

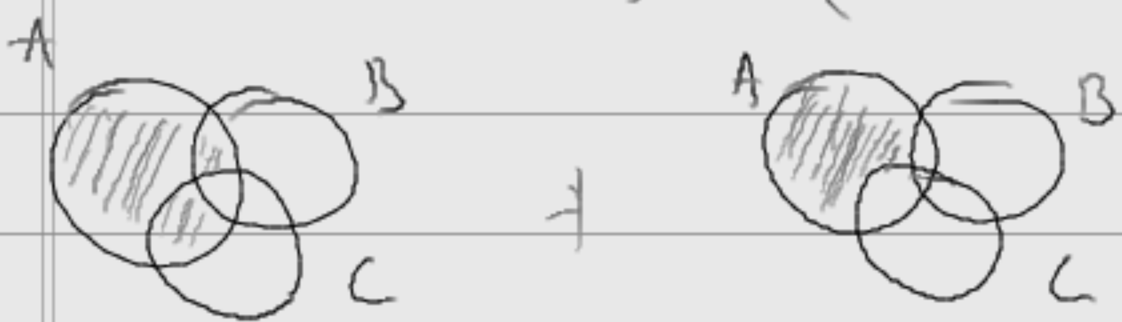
DISPROVING P IS SAME AS PROVING $\neg P$

EX

CONJECTURE:

IF A, B, C ARE SETS THEN

$$A - (B \cap C) = (A - B) \cap (A - C)$$



COUNTEREXAMPLE $A = B = \{1\}, C = \emptyset$

$A - (B \cap C) = \{1\}$, $(A - B) \cap (A - C) = \emptyset$