

Recall that supremum of a set X , denoted by $\sup(X)$, is the smallest upper bound on X . It satisfies that $\forall x \in X, x \leq \sup(X)$ and $\neg \exists y, y < \sup(X) \wedge \forall x \in X, x \leq y$.

Infimum is the largest lower bound.

1: Decide if supremum, infimum, maximum, and minimum of the following sets exist and if yes, find them:

- interval $(2, 5)$

- interval $(-3, 10]$

- $\{\frac{1}{n} : n \in \mathbb{N}\}$

2: Let $x, y \in \mathbb{R}$ and $\varepsilon > 0$. Show that $|x - y| < \varepsilon$ if and only if $x - \varepsilon < y < x + \varepsilon$.

3: Let $x, y \in \mathbb{R}$. Prove triangle inequality. That is $|x + y| \leq |x| + |y|$.