MATH304 HW 4

due **Sep 24** before class, **answer without justification will receive 0 points**. The solution has to be typed (using $\LaTeX$).

1: Show that Erdős-Szekeres Theorem is tight. That is, find a sequence of n^2 number (could be integers) that does not contain a monotone subsequence of length $n+1$.

2: *(P. 83, #12)* Read and try to understand Chinese remainder theorem. Show by example that the conclusion of the Chinese remainder theorem (Application 6) need not hold when m and n are not relatively prime.

3: *(P. 84, #19)*
(a) Prove that of any five points chosen within a equilateral triangle of side length 1, there are two whose distance apart is at most $\frac{1}{2}$.

(c) Determine an integer m_n such that if m_n points are chosen within an equilateral triangle of side length 1, there are two whose distance apart is at most $1/n$.

4: The line segments joining 9 points are arbitrarily colored red or blue. Prove that there must exist three points such that the three line segments joining them are all red, or four points such that the six line segments joining them are all blue (that is, $r(3,4) \leq 9$).

5: *(P. 85, #24)* Let x and t be positive integers with $x \geq t$. Determine the Ramsey number $r_t(t, t, x)$.
(See page 82 for definition of r_t.)

6: *(P. 85, #27)* A collection of subsets of $\{1, 2, \ldots, n\}$ has the property that each pair of subsets has at leas one element in common. Prove that there are at most 2^{n-1} subsets in the collection.