MATH304 HW 11

due **Nov 19** before class, **answer without justification will receive 0 points**. The typing the HW in LaTeX is optional.

1: Determine the generating function for the number h_n of nonnegative integral solutions of

$$2e_1 + 5e_2 + e_3 + 7e_4 = n.$$

(Do not solve h_n, just create the generating function)

2: For every $n \geq 0$ determine the coefficient at x^n in $(1+x)^2(1+x^2)^2(1+x^4)^2(1+x^8)^2\cdots$ which is equal to $\prod_{i=0}^{\infty}(1+x^{2^i})^2$.

3: Find a(n ordinary) generating function where the coefficient of x_k is the number of integers between 0 and $999,999$ whose sum of digits is k.

4: Let α be a real number. Let the sequence $h_0, h_1, h_2, \ldots, h_n, \ldots$ be defined by $h_0 = 1$, and $h_n = \alpha(\alpha-1)\cdots(\alpha-n+1), (n \geq 1)$. Determine the exponential generating function for the sequence.

5: **Using exponential generating function**, find the number of ways to put 30 labeled (thus distinct) people into four different rooms A, B, C, D if room A must have an even number of people (possibly 0) and the other rooms must have at least one person. *(Solution without using exponential generating series will automatically get score 0.)*

6: Find the number of n-digit numbers with digits 1,2,3, and 4 that have odd number of 1s and even number of 2s. Use exponential generating functions.