Chapter 2.1 - Basic Counting Principles

How to count a size of a set S:

- **Addition principle**: Let $S_1, \ldots, S_k$ be disjoint and $S = \cup_{i=1}^{k} S_i$. Then $|S| = \sum_{i=1}^{k} |S_i|$.

- **Multiplication principle**: If $S = (a, b)$, where $a \in S_1$ and $b \in S_2$, then $|S| = |S_1| \cdot |S_2|$.

- **Substraction principle**: If $S \subset U$, then $|S| = |U| - |U \setminus S|$.

- **Division principle**: If S is a disjoint union of sets $S_1, \ldots, S_k$, where $|S_1| = |S_2| = \ldots = |S_k|$, then $|S_i| = \frac{|S|}{k}$.

1: How many ways are there to form a three-letter sequence using the letters a,b,c,d,e,f?
(a) with repetition of letters allowed?
(b) without repetition of any letter?
(c) without repetition and containing the letter e?
(d) with repetition and containing the letter e?

2: A rumor is spread randomly among a group of 10 people by successively have one specified person call someone, who calls someone etc. A person can pass a rumor to anyone except the person who just called and him/herself.
(a) How many different paths can a rumor travel through the group in three calls? n calls?
(b) What is the probability that if A starts the rumor, A received the third call?

3:   Until recently, area codes (area code is a 3 digit number) were created with the following rules

(1) The first digit cannot be a 0 or 1

(2) The second digit must be a 0 or 1

In 1995 this was abandoned when 360 was used in parts of western Washington state (0 still can't be the first number). Q: How many new area codes were made available after 1995?

4:   How many two-digit numbers have distinct and nonzero digits?

5:   How many odd numbers between 1000 and 9999 have distinct digits?