

## Chapter 2.2 - Permutations

Permutation on  $[n]$  is a bijection  $\pi[n] \rightarrow [n]$ .

Permutation is an ordering of a set.

All permutations of set  $\{a, b, c\}$  are  $abc, acb, bac, bca, cab, cba$ .

Example: What is the number of shuffling of a deck of 52 card?

$r$ -permutation is an ordering of  $r$  elements out of a set  $S$  (not all are used).

Number of  $r$ -permutations of an  $n$ -element set is denoted by  $P(n, r)$ .

Claim:  $P(n, r) =$  .

**1:** How many starting puzzles are possible (although not all are solvable) in 15-puzzle? (A grid  $4 \times 4$  with one entry empty and others from 1 to 15).

**2:** What is number of orders of 26 letters from alphabet such that no two vowels (A,E,I,O,U,Y) occur consecutively and letters B and L are next to each other? (it means either BL or LB)?

**3:** Count 7 digit numbers from digits  $\{1, 2, \dots, 9\}$  where 5 and 6 are not consecutive. Digits may not repeat.

Circular permutations: How to arrange  $n$  children in a circle? (symmetric under rotation, but not under flip)

Theorem: 2.2.2: The number of circular  $r$ -permutations of a set of  $n$  elements is  $\frac{P(n,r)}{r} = \frac{n \cdot (n-1) \cdots (n-(r-1))}{r}$ .

**4:** Ten people, including two who don't want to sit next to one another are seated at a round table. How many circular arrangements are there?