Fall 2015, MATH-304

Chapter 3.1 Pigeonhole principle

Theorem 3.1 If $n+1$ objects are distributed to n boxes, then there is a box with at least 2 objects.

Proof: By contradiction. If every box has at most 1 object, then there are at most n objects.

- non-constructive
- also know as Dirichlet's principle
- if there are n boxes, n objects and no box is empty, then every box has exactly one object
- if there are n boxes, n objects and no box has more than 1 object, then every box has exactly one.

Example: Among 13 people are at least 2 who were born in the same month.

1: There are n married couples. How many of the $2n$ people must be selected to guarantee that a married couple is selected?

2: *Section 3.4 Q4* Show that if $n+1$ integers are chosen from $\{1, 2, ..., 2n\}$ then there are always two which differ by 1.

3: *Application 4 from Section 3.1* A chess master has 11 weeks to prepare for a tournament. He plays at least one game per day, but not more than 12 games during any calendar week. Prove there is a succession of consecutive days where the master plays exactly 21 games.

4: *Section 3.4 Q10* A student watches TV at least one hour each day for 7 weeks but not more than 11 hours in any one week. Prove there is some period of consecutive days where the child watches exactly 20 hours of TV. (Assume a whole number of hours of TV watched each day.)

5: *(Application 5 of Section 3.1)* From the integers $1, 2, \ldots, 200$ we choose 101 integers. Show that two of the chosen integers have the property that one divides the other.

6: In the past thousand years, YOU had ancestors A and P such that P was an ancestor to both the father and mother of A. (That is, no-one's family tree is really a tree.) [You can make some assumptions about the total population of the world etc.]

7: Let W be a string of length n on 0 and 1. i.e. $W \in \{0,1\}^n$. Let there be two substrings A, B of W such that $A = B$ and they are disjoint in W. In other words, there exists a substring X of W such that there is a copy of X also in $W - X$. Show that there is always a substring A such that $|A| \geq n/3 - c$ where c is a small constant.
Sidenote: Clearly, $|A| \leq n/2$. It is actually true that $A \geq n/2 - o(n)$ but it is a bit more complicated.