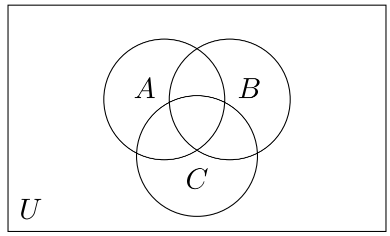
Fall 2015, MATH-304

Chapters 6.1, 6.2 Principle of Inclusion and Exclusion



$|A \cup B \cup C| = |A| + |B| + |C| + \cdots$

1: Find number of integers x such that $1 \le x \le 1000$ and x is not divisible by $5, 6, 8$.

Theorem 6.1.1 Let S be a set and $A_1, A_2, \ldots, A_m \subseteq S$. Then

$$|\overline{A_1} \cap \overline{A_2} \cap \cdots \cap \overline{A_m}| = |S| - \sum |A_i| + \sum |A_i \cap A_j| + \cdots + (-1)^m (A_1 \cap A_2 \cap \cdots A_m).$$

2: Prove Theorem 6.1.1

Corollary 6.1.2

$$|A_1 \cup A_2 \cup \cdots \cup A_m| = \sum |A_i| - \sum |A_i \cap A_j| + \cdots + (-1)^{m+1} \left(A_1 \cap A_2 \cap \cdots A_m\right).$$

3: There are 30 video game players. 15 of them play Legend of Zelda, 17 of them play Call of Duty and 20 of them play World of Warcraft. Legend of Zelda and Call of Duty is played by 8 players, Call of Duty and World of Warcraft is played by 10 players and World of Warcraft and Legend of Zelda also by 10 players. How many players play all three games?

4: Find the number of integers between 100 and 999 inclusive that are not divisible by 4, 6, or 9.

5: Determine the number of solutions of the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 16$ in positive integers x_1, x_2, x_3, x_4 and x_5 not exceeding 7.

6: What is the number of ways to place four nonattacking rooks on the 4-by-4 boards without forbidden positions are marked by $\times$?

		$\times$	
$\times$	$\times$	$\times$	
	$\times$		$\times$

7: I have four exams to study for. I have 21 days to do it. I will study for any given exam during 10 days. For any given pair of exams, I will study them on the same day 5 times. For any given three exams, I will study them together on at most 3 of the days. Finally, I need to “relax” for 3 days (when I don’t study at all). How many days do I have to spend studying for all four exams together?