Fall 2015, MATH-304

Chapters 7.1 Fibonacci Numbers

Study sequences of numbers $h_0, h_1, h_2, \ldots$. Questions: encoding, computing h_n, combinatorial interpretation

Examples of sequences:

- 1 1 1 1 $\cdots$
  $h_0 = 1,\ h_1 = 1,\ h_2 = 2,\ \ldots$
- 0 C $2C$ $3C$ $4C$ $\cdots$
  $h_n = h_{n-1} + C$ or $h_n = nC$
- $D_0\ D_1\ D_2\ D_3\ \cdots$
  $D_n = (n-1)(D_{n-1} + D_{n-2})$, no expression $D_n =$ something.

1: Let h_n be the number of regions in the plane cut by n lanes. Give a formula for h_n.

Fibonacci sequence:
$f_0 = 0,\ f_1 = 1,\ f_n = f_{n-1} + f_{n-2}$

$$0\ 1\ 1\ 2\ 3\ 5\ 8\ \cdots$$

2: Prove that

$$\sum_{i=0}^{n} f_i = f_{n+2} - 1$$

Hint: Induction on n.

3: Prove that f_n is even iff n is divisible by 3.

Solving the Fibonacci recurrence:
Guess f_n behaves like q^n for some q. Recall

$$f_n - f_{n-1} - f_{n-2} = 0 \tag{1}$$

4: Compute candidates for q by replacing f_n by q^n in (1).

5: How to pick the right q for f_n?

6: How many ways are there to tile a $2 \times n$ board using dominoes?

Next time: Chapter 7.2