**MATH-566 HW 2**

Due **Sep 9** before class. Just bring it before the class and it will be collected there.

**1:** (*Carathédory's theorem*)
Let $X \subset \mathbb{R}^d$ and let $\mathbf{x}$ be in the convex hull of $X$. Show that $\mathbf{x}$ is a convex combination of at most $d+1$ points of $X$.
*(Hint: You can use other theorems, like Radon's.)*

**2:** (*Convex sets*)
Construct 4 convex sets $C_1, C_2, C_3$ and $C_4$ in the plane such that the intersection of each 3 of them contains a segment of unit length, but there is no segment of unit length in the intersection of all four of them.

**3:** (*SAGE: convex and affine combination*)
Write a program in sage that will compute if a point $\mathbf{x}$ is a convex/affine combination of given points. The sage template with instructions is provided separately.

**4:** (*Largest disk in a convex polygon*)
Show that it is possible to use linear programming to find the largest disk that can be inscribed in a convex polygon. Suppose the polygon is given as $H$-polytope and formulate a linear program whose solution is the largest disk.
*(It can be done for $H$-polyhedron in higher dimension too, but do just 2D.)*

**5:** (*$V$-polytopes and $H$-polytopes*)
Let $P$ be a $V$-polytope. Show that $P$ is also $H$-polytope.

**6:** (*Polytopes and dual*)
Let $P$ be a 4-dimensional hypercube. That is, $P = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_i \in [-1, 1], i = 1, 2, 3, 4\}$. Describe its dual $P^* := \{\mathbf{y} \in \mathbb{R}^4 : \mathbf{x}^T\mathbf{y} \leq 1 \text{ for all } \mathbf{x} \in P\}$.