

## MATH-566 HW 4

Due **Sep 23** before class. Just bring it before the class and it will be collected there.

### 1: (*Separation of sets*)

(a) Give an example of bounded convex sets  $C$  and  $D$  in  $\mathbb{R}^d$ , where  $C \cap D = \emptyset$  but there are no hyperplane **strictly** separating  $C$  and  $D$ . That is  $\mathbf{a} \in \mathbb{R}^d$  and  $b \in \mathbb{R}$  such that

$$\mathbf{a}^T \mathbf{x} > b, \forall \mathbf{x} \in C \quad \text{and} \quad \mathbf{a}^T \mathbf{x} < b, \forall \mathbf{x} \in D.$$

(b) Give an example of closed convex sets  $C$  and  $D$ , where  $C \cap D = \emptyset$  that cannot be **strictly** separated.

### 2: (*Dualization*)

Dualize your diet problem. Take data of your diet problem from HW1 and create dual of the program and solve it. Answer should contain solution of the dual and interpretation of the result of the dual (what do the numbers mean).

### 3: (*Integer program*)

Paper mill manufactures rolls of paper of a standard width 3 meters. But customers want to buy paper rolls of shorter width, and the mill has to cut such rolls from the 3 m rolls. One 3 m roll can be cut, for instance, into two rolls 93 cm wide, one roll of width 108 cm, and a rest of 6 cm (which goes to waste). Let us consider an order of

- 97 rolls of width 135 cm,
- 610 rolls of width 108 cm,
- 395 rolls of width 93 cm, and
- 211 rolls of width 42 cm.

What is the smallest number of 3 m rolls that have to be cut in order to satisfy this order, and how should they be cut?

(Hints:  $1m = 100cm$ , you may need to consider that variables are integers)

*instead of just real numbers. APMonitor can do it by prefix int in variable and Sage by using `new_variable(integer=True)`).*