MATH-566 HW 6

Due **Oct 7** before class. Just bring it before the class and it will be collected there.

1: (*Simplex method test*)
Use simplex method on the following program:

$$(P)\begin{cases} \text{maximize} & x_1 \\ \text{subject to} & x_1 - x_2 \leq 1 \\ & -x_1 + x_2 \leq 2 \\ & x_1, x_2 \geq 0 \end{cases}$$

What is happening in the computation?

2: (*Ellipsoid method for solving linear programs*)
How would you solve a program (P) = maximize $\mathbf{c}^T\mathbf{x}$ s.t. $A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}$ using the ellipsoid method with an $\varepsilon > 0$ error?
(Suppose both (P) and its dual (D) are superconsistent.)
(Hint: How to formulate the linear program as finding a point in a polytope? Use dual program and ε to guarantee full dimension.)

3: (*Analytic center*)
Let S be defined as intersection of halfspaces $x_i \geq 0$ and $(1 - x_i)^k \geq 0$. Suppose $i \in \{1, 2, \ldots, d\}$ and $k \geq 1$ is odd. Compute the analytic center of S. Notice that for x satisfying $(1 - x_i)^k \geq 0$, the function $(1 - x_i)^k$ is convex.

4: (*Central path*)
Compute central path for the following problem

$$(P)\begin{cases} \text{minimize} & -x_1 \\ \text{subject to} & x_1 \leq 1 \\ & x_2 \leq 1 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{cases}$$

and find the optimal solution using the central path. Plot (sketch) the set of feasible solutions and the computed central path.