MATH-566 **HW 11**

Due **Nov 20** before class. Just bring it before the class and it will be collected there.

1: (*Strength of integer programming*)
Show that in integer program, it is possible to express the following constraint:

$$x \in [100, 200] \cup [300, 400]$$

in other words

$$100 \le x \le 200 \text{ or } 300 \le x \le 400$$

How to express the constraint *without* using *or*?
Hint: use additional integer variable $z \in \{0, 1\}$, consider z and $(1 - z)$.

2: (*What is unimodular?*)
Determine which of the matrices below are (i) unimodular, (ii) totally unimodular, or (iii) neither. Be sure to explain your answer.

$$\underset{\textbf{a.}}{\begin{pmatrix} 1 & -1 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}} \qquad \underset{\textbf{b.}}{\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}} \qquad \underset{\textbf{c.}}{\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}}$$

3: (*Unimodular and totally unimodular*)
Show that $A \in \mathbb{Z}^{m \times n}$ is totally unimodular iff $[A\ I]$ is unimodular (where I is $m \times m$ unit matrix).

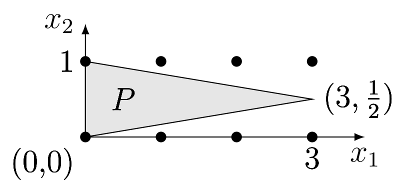
4: (*Integrality for one* **b** *does not imply unimodularity*)
Give an example of an integer matrix A and an integer vector $\mathbf{b}$ such that the polyhedron $P := \{\mathbf{x} \in \mathbb{R}^n | A\mathbf{x} \le \mathbf{b}\}$ is integer, while A is not totally unimodular.

5: (*Branch and bound*)
Solve the following problem using branch and bound. Draw the branching tree too.

$$(P) = \begin{cases} \text{maximize} & -x_1 + 4x_2 \\ \text{subject to} & -10x_1 + 20x_2 \le 22 \\ & 5x_1 + 10x_2 \le 49 \\ & x_1 \le 5 \\ & x_i \ge 0, x_i \in \mathbb{Z} \text{ for } i \in \{1, 2\} \end{cases}$$

You can use any linear programming solver for solving the relaxations.

6: (*Cutting planes*)
Let P be a convex hull of $(0,0),(0,1),(k,\frac{1}{2})$. Give an upper bound on Chvátal's rank of P. (Show it is at most $2k$, actually, it is exactly $2k$.)
Hints: Write P as an intersection of half-spaces, use induction *on k. See what we were doing in notes.*
Drawing of P for $k = 3$.



7: (*Bonus - not graded since Jephian asked in the class.*)
Find a unimodular matrix A, that is not totally unimodular.