# Chapter 3 - Separation theorem
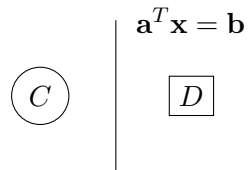
How to show that two convex sets are disjoint?

**Theorem 1.** *Let $C, D \subseteq \mathbb{R}^d$ are convex sets and $C \cap D = \emptyset$ then there exists a hyperplane separating $C$ and $D$. That is, exists $\mathbf{a} \in \mathbb{R}^d$, $b \in \mathbb{R}$ such that*
$\forall \mathbf{x} \in C, \mathbf{a}^T \mathbf{x} \leq b$
$\forall \mathbf{x} \in D, \mathbf{a}^T \mathbf{x} \geq b$

Separation can be strict if $C$ and $D$ closed and one bounded.

$$\mathbf{a}^T \mathbf{x} = \mathbf{b}$$

$C$ $\boxed{D}$

**1:** Why is the theorem true if $C$ and $D$ are compact?

**2:** Why is the theorem true if $C$ compact and $D$ closed?

**3:** Why is the theorem true in general?