Fall 2015, MATH-566

Chapter 3 - Introduction to the Duality for Linear Programming

Let

$$(P)\begin{cases} \text{maximize} & 2x_1 + 3x_2 \\ \text{s.t.} & 4x_1 + 8x_2 \leq 12 \\ & 2x_1 + x_2 \leq 3 \\ & 3x_1 + 2x_2 \leq 4 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{cases}$$

1: Without solving (P) itself, is it possible to provide an upper bound on the value of (P) by using equation $4x_1 + 8x_2 \leq 12$?

2: Without solving (P), is it possible to provide an upper bound on the value of (P) using equations $4x_1 + 8x_2 \leq 12$ and $2x_1 + x_2 \leq 3$?

3: Without solving (P), how would you try to find the combination of constraints that provides the best upper bound? (solution might be another linear program, call it (D))

- (D) gives an upper bound on (P)
- (P) gives a lower bound on (D)

4: Are solutions $\mathbf{x} = (\frac{1}{2}, \frac{5}{4})$ of (P) and $\mathbf{y} = (\frac{5}{16}, 0, \frac{1}{4})$ for (D) optimal solutions?

5: Find the dual program (D) to

$$(P) \begin{cases} \text{maximize} & \mathbf{c}^T\mathbf{x} \\ \text{s.t.} & A\mathbf{x} \le \mathbf{b} \\ & \mathbf{x} \ge 0 \end{cases}$$

6: Find the dual program (D) to

$$(P) \begin{cases} \text{minimize} & \mathbf{c}^T\mathbf{x} \\ \text{s.t.} & A\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \ge 0 \end{cases}$$