Shortest path - and linear programming

Suppose every edge has an orientation (direction). This gives a directed graph. Normal graph can be converted to directed by adding two opposite edges. Note: Previous algorithms work on directed graphs.

1:   Create a linear program solving the shortest path problem. Hints: Minimize, overall cost, for every edge decide if it is in the path or not, make sure that the path starts at s (and ends at t). Make sure that the path does not stop at any other vertex (use that edges are oriented and you know incoming and leaving edges).

2:   Write the linear program for graph with directed edges $E = \{su, sv, uv, ut, vt\}$, where the costs are $c(su) = 2, c(sv) = 5, c(uv) = 1, c(ut) = 6, c(vt) = 3$.

3:   Write the dual linear program for shortest path.

4:   Interpret the dual program.