Fall 2015, MATH-566

Integer Programming - Solution *Methods* - Branch and Bound

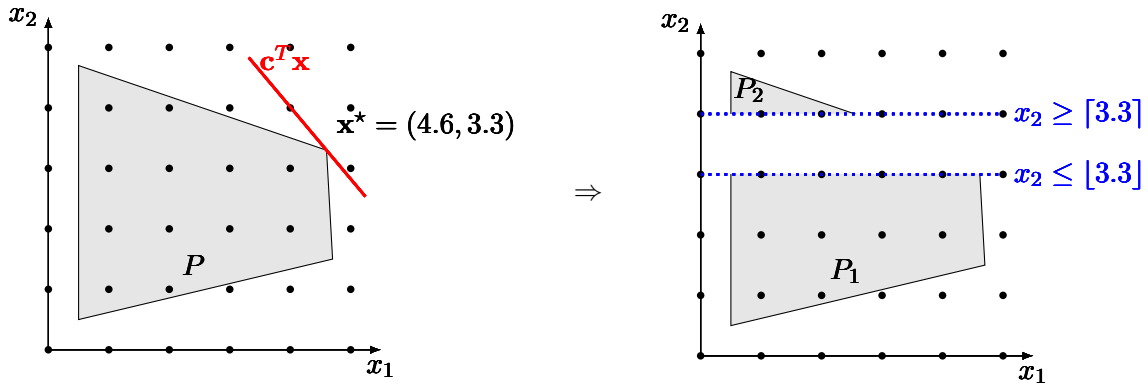
Source: `http://co-at-work.zib.de/files/Gurobi_MIP.pdf`

Problem:

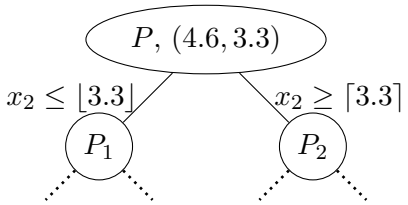
$$(IP)\begin{cases}\text{maximize} & \mathbf{c}^T\mathbf{x} \\ \text{subject to} & A\mathbf{x} \le \mathbf{b},\end{cases}$$

where $\mathbf{c} \in \mathbb{Z}^n, \mathbf{b} \in \mathbb{Z}^m, A \in \mathbb{Z}^{m\times n}$, and $\mathbf{x} \in \mathbb{Z}^n$.

Suppose we try to relax the problem and solve it as a linear programming problem. The set of feasible solutions is P. Suppose that the optimum is $\mathbf{x}^\star = (4.6, 3.3)$. We know x_2 cannot be 3.3. So we create two new instances, where we add constraints $x_2 \ge \lceil 3.3 \rceil$ and $x_2 \le \lfloor 3.3 \rfloor$. Variable x_2 is *branch variable*. We solve them both and better of the solutions is solution to the original problem.



The same process repeats with P_1 and P_2. Result is a *big* branch and bound tree T.



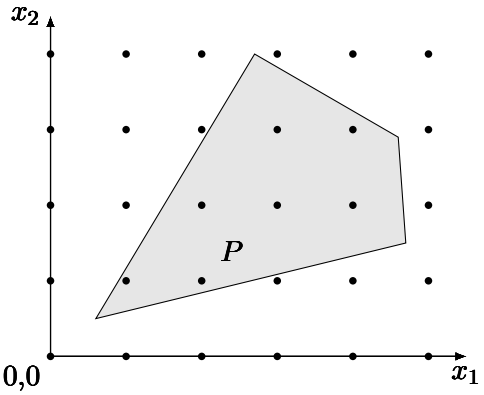
Branch and (no Bound) outline

1. Let $P = \{\mathbf{x} : A\mathbf{x} \le \mathbf{b}\}$
2. Build tree T with one node P (and mark it unexplored)
3. while T has unexplored node X
4. $\quad\quad$ $\mathbf{x}^\star :=$ optimum for LP relaxation of X; mark X explored
5. $\quad\quad$ If $\mathbf{x}^\star_i \notin \mathbb{Z}$ for some i
6. $\quad\quad\quad\quad$ $X_1 := X \cap \{\mathbf{x} : \mathbf{x}_i \le \lfloor \mathbf{x}^\star_i \rfloor\}$
7. $\quad\quad\quad\quad$ $X_2 := X \cap \{\mathbf{x} : \mathbf{x}_i \le \lceil \mathbf{x}^\star_i \rceil\}$
8. $\quad\quad\quad\quad$ Add X_1 and X_2 to T as unexplored nodes
9. Return maximum of integer solutions in T.

1: Consider problem

$$(IP)\begin{cases}\text{maximize} & 100x_2 + x_1 \\ \text{subject to} & (x_1, x_2) \in P,\end{cases}$$

where P is depicted below. Solve (IP) using Brand and Bound. Create branch and bound tree T.



2: Will branch and bound ALWAYS find an optimal solution?

3: Is there a *good* bound on the size of the tree?

4: Is it possible to identify nodes in T that will not contain the optimal solution?

5: What are (dis)advantages of processing nodes deep in the search tree vs nodes close to the root?

6: Which if a solution in a node has more non-integer coordinates, which variable to branch on first?

Next time: Cutting Planes for Integer Programming.