MATH314 HW 7

due **Mar 10** before class, **answer without justification will receive 0 points**. The typing the HW in LaTeX is optional.

If question has (No drawing), you must presents a writeup that is complete and correct without using a picture. If you add a figure to (No drawing) question, it will not be treated negatively but you should not refer to it in the solution.

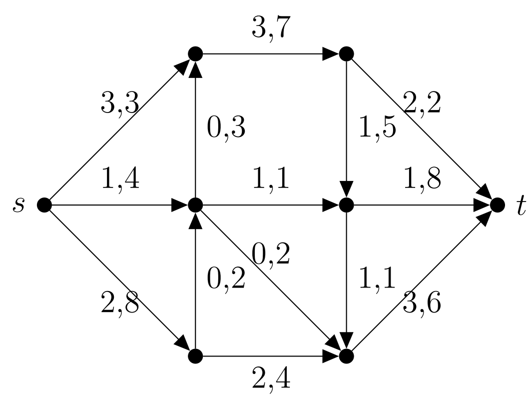
1: Let G be a k-connected graph and let S be any set of k vertices. Show that if a graph H is obtained from G by adding a new vertex w and joining w to the vertices of S, then H is also k-connected.

2: Let G be a k-connected graph of order $n \geq 2k$ and let U and W be two disjoint sets of k vertices of G. Prove that there exist k disjoint paths connecting U and W.

3: Use Theorem 5.21 or 5.22 to show that $\kappa(G) = \lambda(G)$ when G is 3-regular. (I'm asking you to reprove Theorem 5.20, but with a different proof. Copying the proof in the book and not using Theorems 5.21 or 5.22 is not an acceptable solution.)

4: Let u and v be two vertices in an oriented graph G. Describe an algorithm to find the maximum number of internally disjoint $u - v$ paths (paths starting at u and ending in v). (Hint: Use network flows. How to make sure every vertex is used only in one path?)

5: Consider the network below with given capacity and flow values. (The edge label f, u means flow-value f and capacity u.) Find a sequence of augmenting paths and augment the flow to a maximum flow.



6: Let (G, u, s, t) be a network, and let $\delta^+(X)$ and $\delta^+(Y)$ be minimum s-t-cuts in (G, u). Show that $\delta^+(X \cap Y)$ and $\delta + (X \cup Y)$ are also minimum s-t-cuts in (G, u).