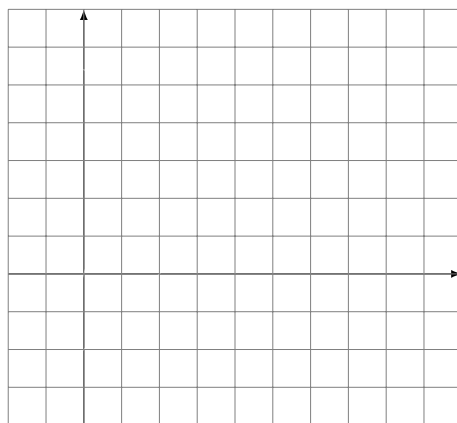


Geometry behind linear programming and basics

Exercise: Solve the following linear program:

$$(LP) \begin{cases} \text{minimize} & x + y \\ \text{s.t.} & x + 2y \leq 14 \\ & 3x - y \geq 0 \\ & x - y \leq 2, \end{cases}$$



Basic linear programming definitions:

- *feasible solution* is vector \mathbf{x} such that $A\mathbf{x} \leq \mathbf{b}$. In other words, point satisfying all the constraints.
- *set of feasible solutions*
- *optimal solution* is a feasible solution that is maximizing/minimizing the objective function.

1: What shape is the set of feasible solutions?

2: What shape is the set of optimal solutions?

3: Construct a linear program that has more than one optimal solution.

4: Construct a linear program that has no feasible solution.

5: Construct a linear program that has no optimal solution.

Basic geometric definitions (for LP in higher dimension): Suppose we live in \mathbb{R}^d for some $d \in \mathbb{N}$.

- *hyperplane* is $d - 1$ dimensional subspace $\{\mathbf{x} \in \mathbb{R}^d : \mathbf{a} \cdot \mathbf{x} = c\}$, where $a \in \mathbb{R}^d$ and $c \in \mathbb{R}$.
- *closed halfspace* is $\{\mathbf{x} \in \mathbb{R}^d : \mathbf{a} \cdot \mathbf{x} \leq c\}$, where $a \in \mathbb{R}^d$ and $c \in \mathbb{R}$.
- $C \subseteq \mathbb{R}^d$ is *convex* if $\forall \mathbf{x}, \mathbf{y} \in C, \forall t \in [0, 1], t\mathbf{x} + (1 - t)\mathbf{y} \in C$. (line between x and y is in C)

Draw figures in class

Note: Intersection of family of convex sets is a convex set (obvious from the definition)

6: Show that the set of feasible solutions to any linear program is a convex set.