

## Separation theorem

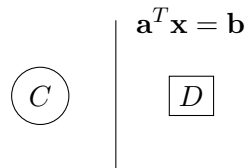
How to show that two convex sets are disjoint?

**Theorem 1.** *Let  $C, D \subseteq \mathbb{R}^d$  are convex sets and  $C \cap D = \emptyset$  then there exists a hyperplane separating  $C$  and  $D$ . That is, exists  $\mathbf{a} \in \mathbb{R}^d, b \in \mathbb{R}$  such that*

$$\forall \mathbf{x} \in C, \mathbf{a}^T \mathbf{x} \leq b$$

$$\forall \mathbf{x} \in D, \mathbf{a}^T \mathbf{x} \geq b$$

Separation can be strict if  $C$  and  $D$  closed and one bounded.



**1:** Why is the theorem true if  $C$  and  $D$  are compact?

**2:** Why is the theorem true if  $C$  compact and  $D$  closed?

**3:** Why is the theorem true in general?