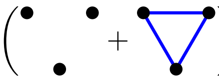
Homework 1

1: Find the largest $c \in [0,1]$ such that

$\phi\Big($  $\Big) \geq c$

for all $\phi \in Hom^+(\mathcal{A}, \mathbb{R})$.

2: By Mantel's theorem, an n-vertex graph with $\left\lfloor \frac{n^2}{4} \right\rfloor + 1$ edges has a triangle (for $n \geq 3$). Show that in fact it has at least $\lfloor \frac{n}{2} \rfloor$ triangles. Hint[1]

3: Show that the number of monochromatic triangles in any 2-coloring of the edges of K_n is at least

$$\frac{n(n-1)(n-5)}{24}.$$

4: [Bondy] Let G be a graph with more than $e(T_k(n))$ edges and maximum degree d, then the neighborhood of every vertex of maximum degree in G contains more than $e(T_{k-1}(d))$ edges.

[1] Adapt the first proof of Mantel's theorem. Separate the case when every edge is contained in a triangle.

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